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RESEARCH PAPER

Application of an improved zonality index model integrated with multivariate fractal analysis: epithermal gold deposits

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Abstract
The main goal of this present study is to apply an improved zonality index to epithermal gold ores based on staged factor and number-size (N-S) fractal analysis. This technique was utilized in Bardaskan district, NE Iran, which is epithermal gold mineralization. An improved zonality index is a proportion of factors including ore and pathfinder elements based on rock samples. Consequently, two factors were selected after four stages of the staged factor analysis which consist on F-4 (As-Fe-Mo-S) and F 3-4 (Au-Ag). Based on these factors, (F1-4)/(F3-4) is determined as an improved zonality index. In addition, the improved zonality index was categorized by fractal modeling. The N-S model demonstrates that the major anomalies of this improved zonality index are associated with silicification as main alteration zone and the intersections of faults, particularly in the NE and northern parts of the Bardaskan region. Additionally, additional rock samples with Au higher than 100 ppb are located in anomalous parts of the improved zonality index and marginal parts of this area with high values of this index contain pathfinders of gold mineralization, especially As based on this methodology. This methodology could be strongly considered in the exploration of different types of mineral deposits and the classification of the target anomalies.

Keywords: Improved Zonality Index; Multivariate Fractal Model; Epithermal Gold.

Introduction

Geochemical anomalies / backgrounds are significant indications to define ore deposits / occurrences in previous phases (reconnaissance and prospecting) of mineral exploration, specifically for buried mineralization and deposits (Chaffee, 1976; Roslyakov, 1984; Carranza & Sadeghi, 2012; Li et al., 2016; Sadeghi, 2020; Koohzadi et al., 2021; Torshizian et al., 2021).

Beus and Grigoryan (1977) and Grigoryan (1992) proposed zonality indexes according to the ratio of main and mobile elements which are termed supra- and sub-ore elements in various ore mineralization. A zonality index is utilized to distinguish first and secondary haloes as exploration keys in the marginal part of a deposit, which is significant for depth exploration and drilling operation. However, this procedure can be applied to distinguish geochemical patterns

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and the relevant anomalies for further exploration (Ziaii et al., 2011; 2019; Imamalipour et al., 2018; Lin et al., 2020; Heidari et al., 2021). Mathematical methods have been used for improved interpretation of zonality index, e.g., multivariate fractal analysis (Aliyari et al., 2020; Heidari et al., 2021).

A multivariate statistical method is a general technique for the decrease of a large number of regionalized variables into a few groups as named factors which include the same variables, such as ore and pathfinder elements (Reimann et al., 2002; Treiblmaier and Filzmoser, 2010; Zuo, 2011; Nazarpour, 2018; Afzal et al., 2016; Hoseinzade & Mokhtari, 2017; Daneshvar Saein, 2021; Sadeghi et al., 2021a, b). Factor analysis in geochemical exploration has been applied for paragenesis classification of ore and trace elements. Moreover, the factor scores are used to model geochemical anomalies / zones (Yousefi & Kamkar-Rouhani, 2012; Sadeghi, 2020; Zissimos et al., 2021). Yousefi et al. (2014) applied staged factor analysis (SFA) to qualify factor scores in porphyry systems. Also, Aliyari et al. (2020) used these methods for the development of the zonality index in an Iranian porphyry system.

Fractal/multifractal analysis has been developed based on fractal geometry (Mandelbrot, 1983) to interpret geochemical data. Different traditional fractal models have been improved and applied for classification of geochemical anomalies and zones (Mandelbrot, 1983; Cheng et al., 1994; Cheng, 1999; Li et al., 2003; Afzal et al., 2011; Yasrebi and Hezarkhani, 2019; Sadeghi, 2021a; Sadeghi & Agterberg, 2021). Different fractal models are proposed based on a relationship between regionalized variables, e.g., ore grades, and their occupied geometrical spaces (Nazarpour, 2018; Agterberg, 1995; Karaman et al., 2021). Numerous fractal models have also been developed recently such as concentration-concentration, concentration-distance from centroids, concentration-distance from faults, category-based, and simulated fractal models which are more associated with the geological properties (Adib et al., 2021; Nabilou et al., 2021; Sadeghi et al., 2015; Sadeghi, 2021a,b; Sadeghi & Cohen, 2021).

In this research, the zonality index is improved by SFA fractal analysis in Bardaskan epithermal system, NE Iran. In addition, the ratio between factors of major and pathfinder elements was estimated and classified by a number-size fractal model for the detection of major prospects. Results derived via this methodology were compared with geological particulars and further samples, for further interpretation.

Methodology

Zonality index

Beus and Grigoryan (1977) established a zonality index for the detection of major minrealization / primary geochemical haloes from secondary dispersion for different mineralization types (Ziaii et al., 2012; 2019; Aliyari et al., 2020). The primary haloes occurred based on penetration of ore-forming fluids in host rocks. The zonality model is an appropriate index for the classification of anomalies or mineralized zones based on a ratio between supra- and sub-ore haloes' elements in various ore deposits (Reimann et al., 2002; Imamalipour et al., 2018; Ziaii et al., 2019). In this research, the target is gold epithermal mineralization. Major elements are Au, and Ag in core of mineralization. However, As, and Sb mineralization exists in marginal parts of a gold epithermal system (Beus & Grigoryan, 1977; Reimann et al., 2002).

Number-size (N-S) fractal model

This model is a traditional and basic fractal analysis to characterize natural features, specifically geochemical characteristics (Mandelbrot Mandelbrot; Agterberg, 1995; Sadeghi et al., 2012; Malaekeh et al., 2021). Concentration-Number (C-N) by Hassanpour and Afzal (2013) and

Simulated Size-Number (SS-N) by Sadeghi et al., (2015) were proposed based on this method. Raw data is used for the N-S model (Jebeli et al., 2018; Afzal et al., 2019; Beyranvand et al., 2021). This model is proposed in the following form (Mandelbrot, 1983):

$$N(\geq\rho) \propto \rho^{-\beta} \quad (1)$$

$N(\geq\rho)$ represents the sample number with concentration values greater than or equal to the ρ values; ρ is the concentration of the target element and β is the fractal dimension. This method can be used for stream sediments, in-situ soils, and rock samples in geochemical exploration (Hashemi & Afzal, 2013; Afzal et al., 2014; Shamseddin Meigoony et al., 2014; 2021; Hosseini et al., 2015; Momeni et al., 2015; Rezaei et al., 2015; Zadmehr and Shahrokhi, 2019; Saadati et al., 2020). Here, this is used for the definition of zonality index zones based on a general and improved zonality model.

SFA

This is an improvement from the stepwise factor analysis (SWFA). In the SWFA, noisy elements are removed. The noisy elements are not placed in any groups (i.e., factors). It is a developed factor analysis for the detection of important multi-elemental signatures of ore deposits. The noisy elements are detected and removed from the analysis until a satisfactory multi-elemental signature is accomplished (Yousefi et al., 2012; Afzal et al., 2016; 2017; Farahmandfar et al., 2020; Sadeghi et al., 2021a).

Furthermore, classical factor analysis with varimax rotation was utilized to extract the usual factors. Varimax rotation is a statistical technique used at one level of factor analysis as an attempt to clarify the relationship among factors. In addition, 0.5 was considered the threshold value for loading to detect factors. In this study, the ratio of factors related to pathfinder and ore minerals was used to calculate the zonality index. In addition, ratio of pathfinder elements factor divided by ore elements factor was applied for evaluation of the improved zonality index. Consequently, this index was categorized using N-S fractal modeling.

Geological setting

The Bardaskan area of (~7.5 km2) is located in Khorasan Razavi province, NE Iran. This area is located in the Taknar zone as an important subdivision of the Iranian central structural zone adjacent to the north Darouneh fault (Alavi, 1994; Hashemi et al., 2010). Metallic ores, e.g., gold and copper consist of epithermal / disseminated mineralization (Hashemi and Afzal, 2013; Daneshvar Saein, 2021). This area consists of Ordovician igneous, metamorphic, and volcanoclastic rock types from Taknar zone. The igneous rocks include rhyodacite, rhyolite, spillite and microgabbro. The metamorphic rocks consist of schists, meta-tuffaceous sandstones, sericite-chlorite schists, and slates (Hashemi & Afzal, 2013; Daneshvar Saein, 2021).

The E-W and NE-SW faults are existed in the Bradaskan region (Fig 1). There are many structures with high density in this area. Main alteration zones for epithermal ores include silicification, sericitization, and chloritization types in this area. Moreover, major ore minerals are chalcopyrite, pyrite and gold particles within silicic vein/veinlets and sericitic alteration zones (Hashemi et al., 2010; Hashemi & Afzal, 2013; Daneshvar Saein, 2021).

Many epithermal gold mineralization / deposits occurred in the Bardaskan, e.g., Sebandoon, Bijvard and Damanghor (Hamami Pour et al., 2014; Abbasnia et al., 2019). The volcano-plutonic rocks specifically rhyolites and rhyodacites are exist in the lower part of the Taknar formation. The volcanoclastic rocks, especially tuffaceous sandstones and also, the metamorphic units which include shists and phyllites, occurred to upper part of Taknar formation.

The vein form mineralization occurred in the fractures / faults as filling of spaces in different parts of this area especially in the northern part. These veins / veinlets are quartz-quartz-sulfide.

Major and minor sulfides are chalcopyrite and pyrite, respectively. This mineralization happened inside the metamorphic tuffaceous sandstones and shists as altered host rocks. Based on geological evidences mentioned above, the Bardaskan gold mineralization looks similar to the epithermal low sulfidation type gold ores. These characteristics include pyrite and chalcopyrite, vein / veinlet texture, silicic and argillic alteration zones, gold, copper and arsenic concentrations (Hashemi, 2010; Hashemi & Afzal, 2013).

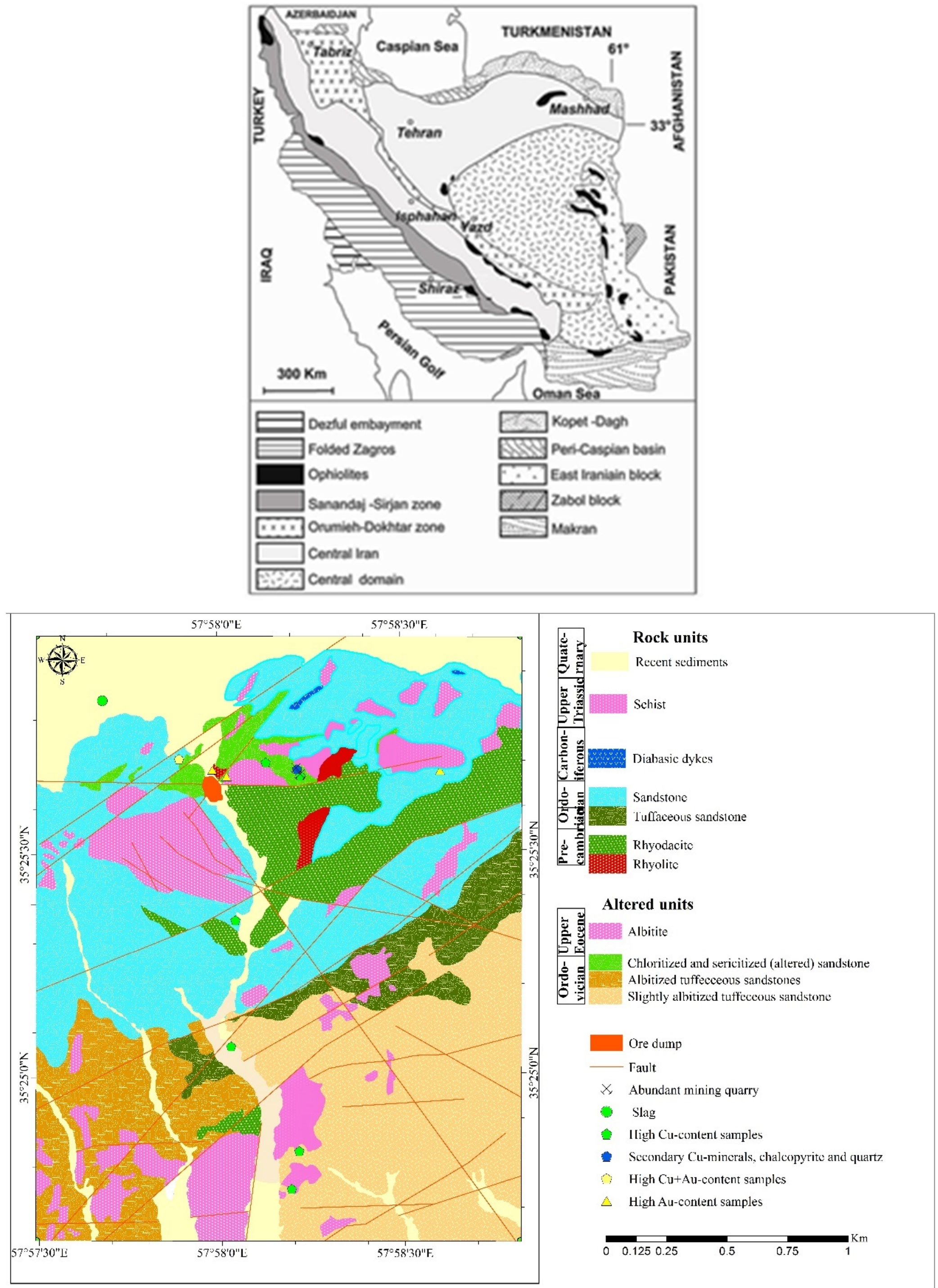


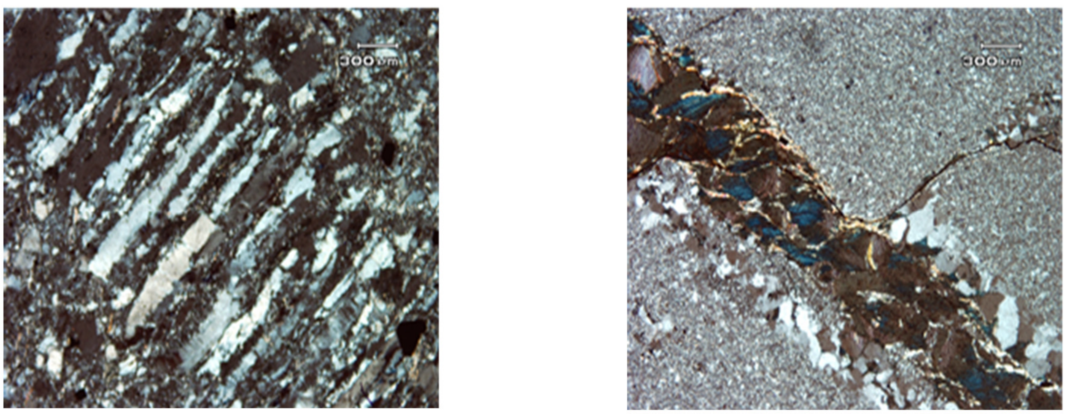
Figure 1. Location of Bardaskan area in structural map of Iran (Alavi, 1994) and geological map of the area (Hashemi & Afzal, 2013)

Alteration

The main alteration zones in this area include silicification, chloritization, sericitization with albitization and carbonatization. Albitization is extended in a variety of areas, specifically at rhyodacites, and it has does have any relationship with mineralization. Carbonatization is existed in the form of vein/veinlet, which can be a sign of low sulfidation epithermal gold deposits (Robb, 2005). Silicification expands in this district, especially in the northern part of Bardaskan with high volume of chloritization zone (Fig 2). In several parts, silicification is existed along with sericitization as shown in Fig. 2. Chloritization and sericitization have highest occupied areas in this region. Plagioclases were formed to sericites in the majority of the igneous rocks in this area (Hashemi, 2010).

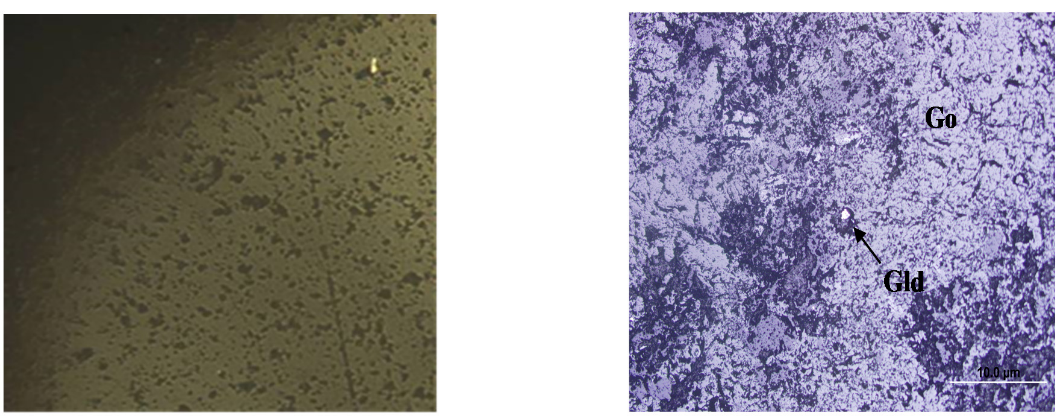
Mineralization

Based on the mineralographical, fluid inclusion and stable isotope studies carried out by Hashemi (2010) in Bardaskan, metallic ores were formed in three stages in-cluding magmatic, hydrothermal (hypogene) and supergene (weathering). In the first stage (magmatic), magnetite and specularite occurred. Gold mineralization was happed in the second stage with pyrite, tetrahedrite, chalcopyrite, galena and sphalerite (Fig 3).



(a) (b)

Figure 2. Alteration zones in thin sections including (a) quartz and chlorite veinlets in meta-sandstone; (b) accumulation of quartz, chlorite and sericite in silty-sandstones



(a) (b)

Figure 3. Gold mineralization defined in polished sections including (a) gold particles (Gld) in Quartz in the second stage; (b) gold particles with goethite (Go) in the supergene stage

The hydrothermal main stage occurred within silicification, as demonstrated in Fig 3. Fi-nally, native copper, goethite, hematite, malachite, azurite, cuprite, tenorite and martite occurred in supergene stage. Quartz, calcite and chlorite are the main gangues in this area, and quartz is the main host for sulfides and gold. Free gold particles and electrum are major gold ores in this area and several sulfides, especially pyrites host gold in silicic veins/veinlets. Several gold particles were indicated in supergene stage as well with goethite, as shown in Fig. 3.

Discussion

Dataset

In this study, 483 rock samples were collected (Fig. 4) and were analyzed using ICP-MS for elements related to the gold epithermal mineralization. Statistical parameters for gold are calculated (Table 1) and also, mean, median, and maximum values are 38 ppb, 18 ppb, and 8 ppm, respectively. Statistical characteristics for gold and related elements are available in. The gold distribution is not normal and has a high standard deviation (Fig. 5).

Table 1. Statistical parameters of Au, Cu, As, and Sb in the Bardaskan area

	Au (ppb)	Cu (ppm)	As (ppm)	Sb (ppm)
Mean	38	437	10.3	1.72
Median	18	41	21	7
SD	357	3000	28.3	1.83
Maximum	8,540	46,730	1,060	39
Minimum	1	1	5	5

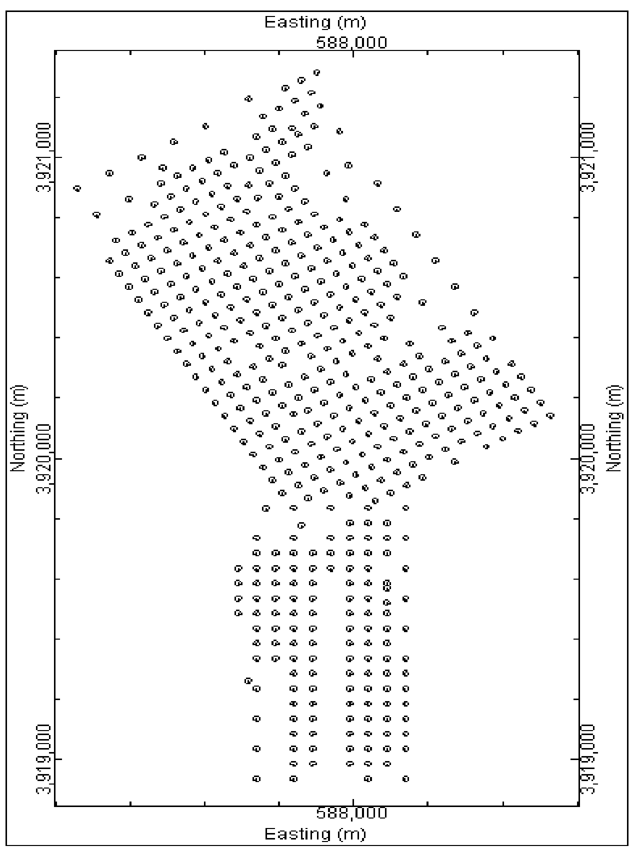


Figure 4. Rock sampling grid in the Bardaskan area

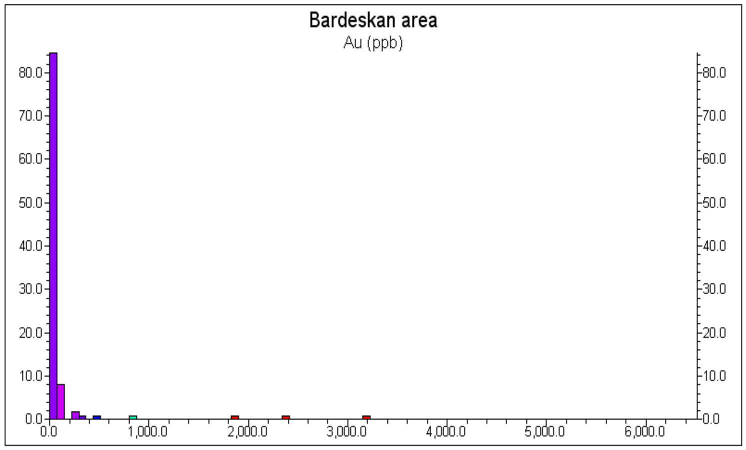


Figure 5. Au histogram in the study area

Multivariate analysis

To reduce the number of the variables, SFA was calculated for 34 elements, then classified using SPSS software. The loading plot at rotated space is indicated in Fig. 6 to rep-resent the extracted factors. The stepwise factor analysis (SWFA) was utilized to the three stages of lithogeochemical data to remove noisy elements. Then, factors were cleaned for the SFA to be applied in the fourth stage. The threshold for noisy elements' scores is 0.5 in the SFA to recognize highly contributing elements in the factors, based on the rotated matrix. Based on SWFA, the noisy elements are Ni, Sb, Co, Pb and Th. Based on epithermal gold mineralization, Au, Ag, As, Cu, Mo, Zn, Fe, S, Bi, Cd and La were selected for the final stage of the SFA (Tables 2; Fig 6). In the end, As, Fe, Mo and S (pathfinders for epithermal gold ore) were grouped in the first factor in the fourth stage (F1-4). Here Au is grouped with Ag in the F3-4 which is factor for main mineralization.

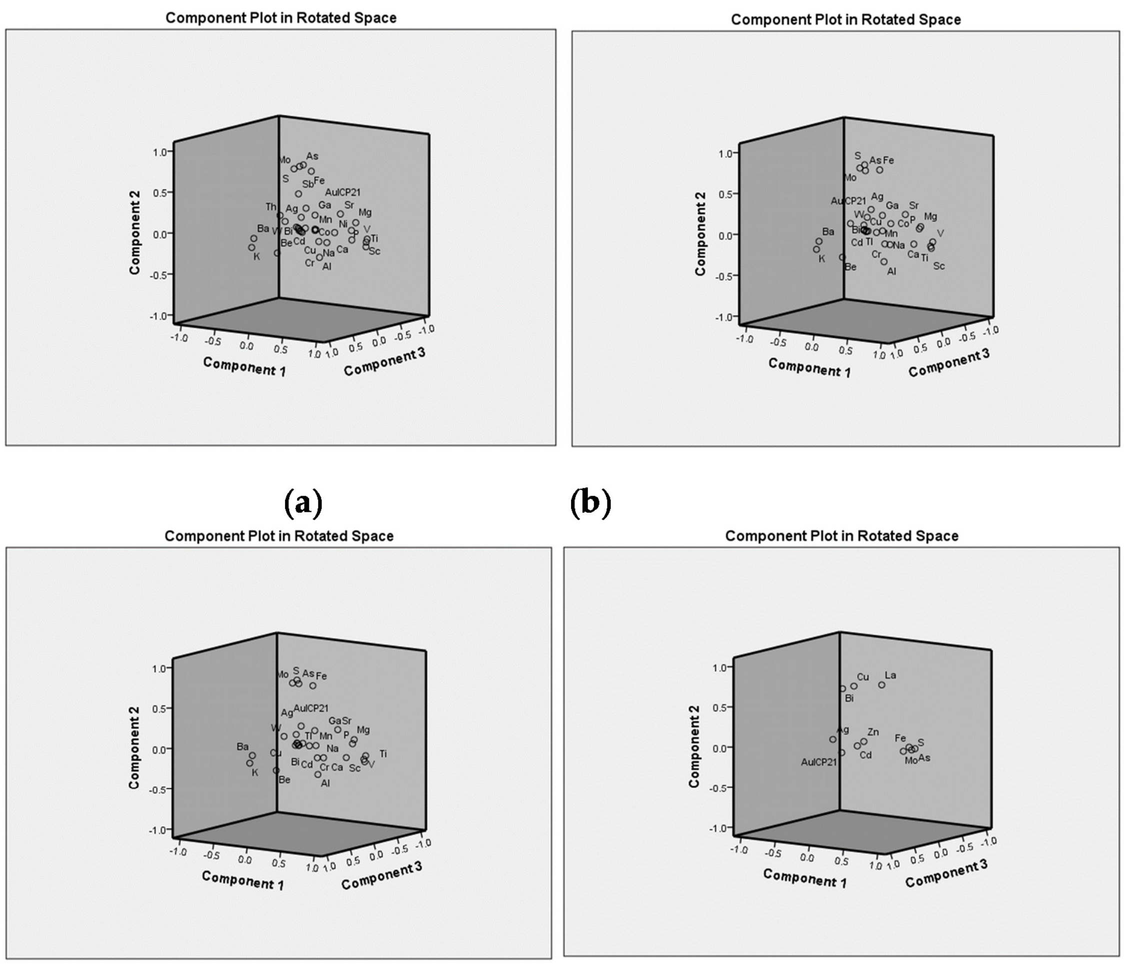
Zonality-multivariate fractal modeling

In this research, the zonality index was calculated by an improvement method based on the results obtained by the SFA. The ratio between two factors related to Supra- and Sub-ore elements in the epithermal mineralization is calculated. This improved index is equaled to (F1-4)/(F3-4) because major elements are Au and Ag and As with S, Fe and Mo are marginal and minor ore elements in the epithermal gold ores. Then, the N-S fractal model was used to this zonality index. The zonality index N-S log-log plot represents five populations as demonstrated in Fig 7. Distribution map of $\frac{F1-4}{F3-4}$ was constructed by the Inverse Distance Squared (IDS) estimation method by a 10 m^2 cell size (Fig 8). Main anomalous parts are equal to lower than 0.4 of $\frac{F1-4}{F3-4}$ data, which are situated in the NE, northern, and small portions in the central parts of this area, as depicted in Fig 6. High values of this ratio based on the fractal modeling (≥ 8.5) as the last population are located in the marginal parts of this region, especially in the western and eastern parts (Fig. 8). Moreover, a conventional zonality index based on $\frac{(As\times Sb)}{(Au\times Ag)}$ was calculated for comparison with results of factors ratios. The N-S fractal model was applied to this index too, as displayed in Fig 7. There are five populations such as improved zonality index and major anomalies (≤ 0.4 of $\frac{(As\times Sb)}{(Au\times Ag)}$) data which are located in the NE to northern, and small portions in the southern parts of this area, as demonstrated in Fig. 8.

Table 2. The final step of the SFA in the Bardaskan area for separation of major and pathfinder factors

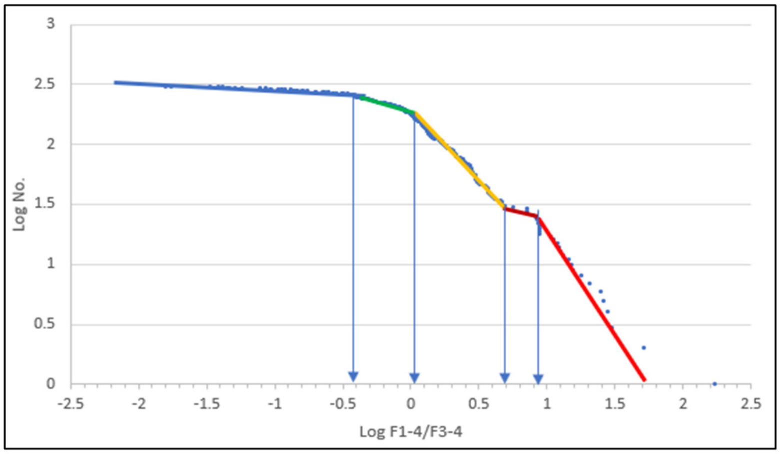
Rotated Component Matrix[a]

	Component			
	1	**2**	**3**	**4**
Ag	.102	.168	**.765**	.231
As	**.745**	.003	-.033	.115
Bi	.001	**.739**	.420	-.053
Cu	.000	**.737**	.180	.029
Fe	**.760**	.043	.110	-.016
La	.057	**.686**	-.320	.012
Mo	**.759**	.009	.230	-.070
S	**.784**	.009	.093	-.031
Zn	.024	.025	.007	**.840**
Au	.220	.013	**.751**	-.087
Cd	-.023	-.025	.077	**.852**

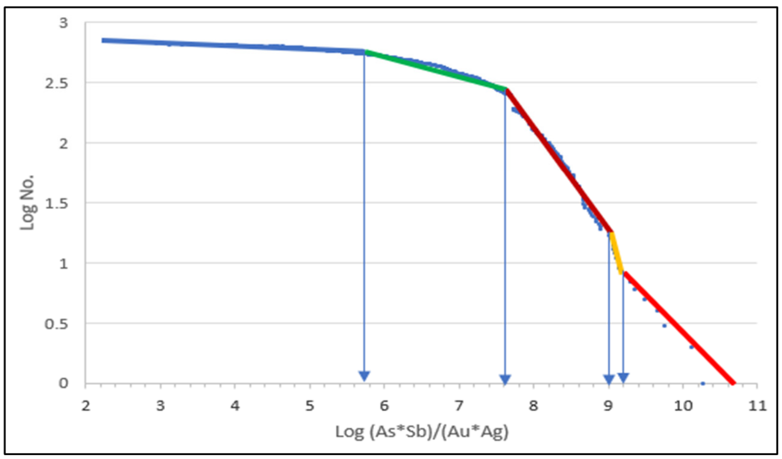


(c) (d)

Figure 6. Loading plots in rotated space for the first (a), second (b), third (c) and fourth (d) stages of the SFA



(a)



(b)

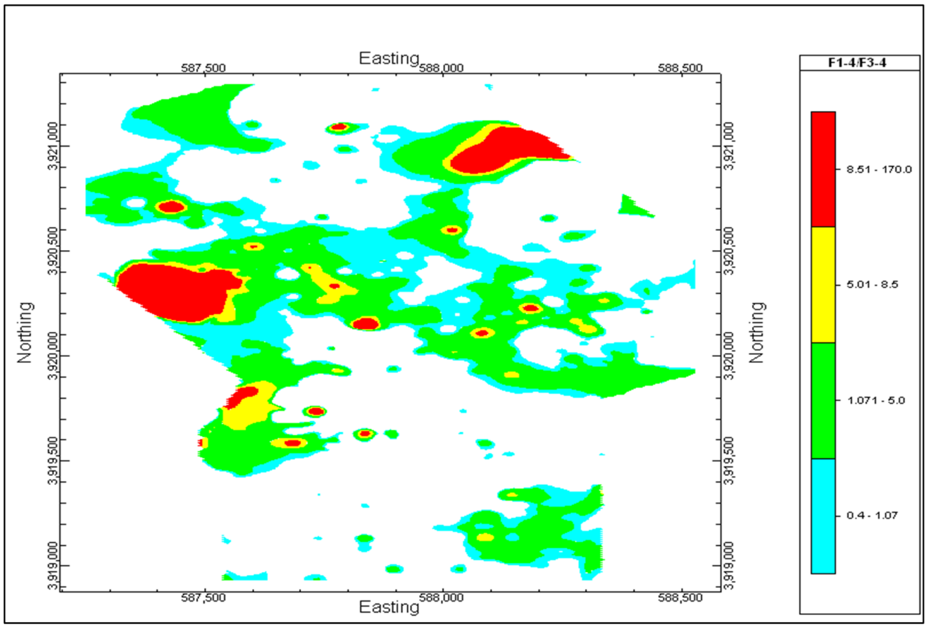
Figure 7. The N-S log-log plots of (a) $\frac{F1-4}{F3-4}$ and (b) and $\frac{(As\times Sb)}{(Au\times Ag)}$

Validation of results with geological particulars/mineralized rock samples

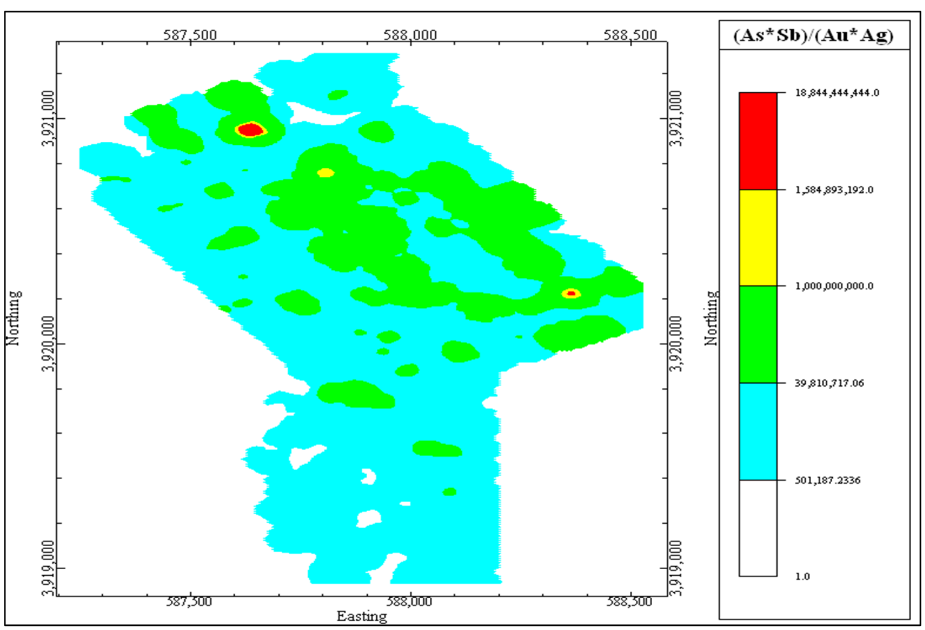
Results derived via the improved zonality index ($\frac{F1-4}{F3-4}$) were compared with geological evidences and mineralized samples (Fig. 9). Further rock samples with Au≥100 ppb are located in the major population of the improved zonality index ($\frac{F1-4}{F3-4}$) ≤0.4). A comparison between the main population of zonality and further rock samples with the gold values higher than 100 ppb was carried out using the overall accuracy (OA) matrix (Carranza, 2011). This matrix is used for the calculation of overlapping between two datasets or models, as depicted in Table 3. The OA is calculated by four matrix elements. Comparison between Au rock samples (≥100 ppb) with the main class from the zonality index, defined by the fractal modeling, shows that the OA is 0.85, showing a good correlation (Table 4).

Major anomalies of the improved zonality index (≤ 0.4) are situated in the NE to northern areas, correlated with second stage of ore mineralization based on gold particles and electrum occurrences at the silicification. Moreover, small portions in the central parts of this area are correlated with supergene stage based on weathering and oxidation evidences such as hematite and goethite. High values of this ratio based on the fractal modeling (≥ 8.5) as the last population in the marginal parts of this area are associated with albitization and chloritization. There are disseminated Au mineralization with low grades.

In addition, the main anomalies of improved zonality index are associated with alteration zones and faults, as depicted in Fig. 8. The strong anomalies of the improved zonality index are associated with chloritization, sericitization, and silicification altera-tion zones (Fig. 10). It shows a good correlation with the improved zonality index with silicification. The silicification in the northern part of this area which is associated with major anomalies. Sericitization is existed in several parts of the north of the study area (Fig. 10).



(a)



(b)

Figure 8. Distribution maps of (a) $\frac{F1-4}{F3-4}$ and (b) and $\frac{(As \times Sb)}{(Au \times Ag)}$ classified using the N-S fractal model

Furthermore, there are a variety of faults and the relevant intersections which are cross-cutting the main population of this zonality index, as shown in Fig. 10. This dense fault system is a positive sign for epithermal gold deposit (Robb, 2005). Based on the above-mentioned evidence, high anomalies of the improved zonality index can be defined for further exploration in this epithermal system.

Table 3. Overall accuracy (OA) matrix procedure (Carranza, 2011)

		Rock samples (Au≥ 100 ppb)	
		Inside anomaly	Outside anomaly
Number of $\frac{F1-4}{F3-4}$ anomalous samples	Inside anomaly	True positive (A)	False positive (B)
	Outside anomaly	False negative (C)	True negative (D)
		Overall accuracy = (A+D)/(A+B+C+D)	

Table 4. The OA between improved zonality index and main anomalous samples for two zonality indexes

		Rock samples (Au≥ 100 ppb)	
		Inside anomaly	Outside anomaly
Number of $\frac{F1-3}{F2-3} \leq 0.4$ high anomalous pixels	Inside anomaly	9	1349
	Outside anomaly	3	7653
		Overall accuracy = 0.85	

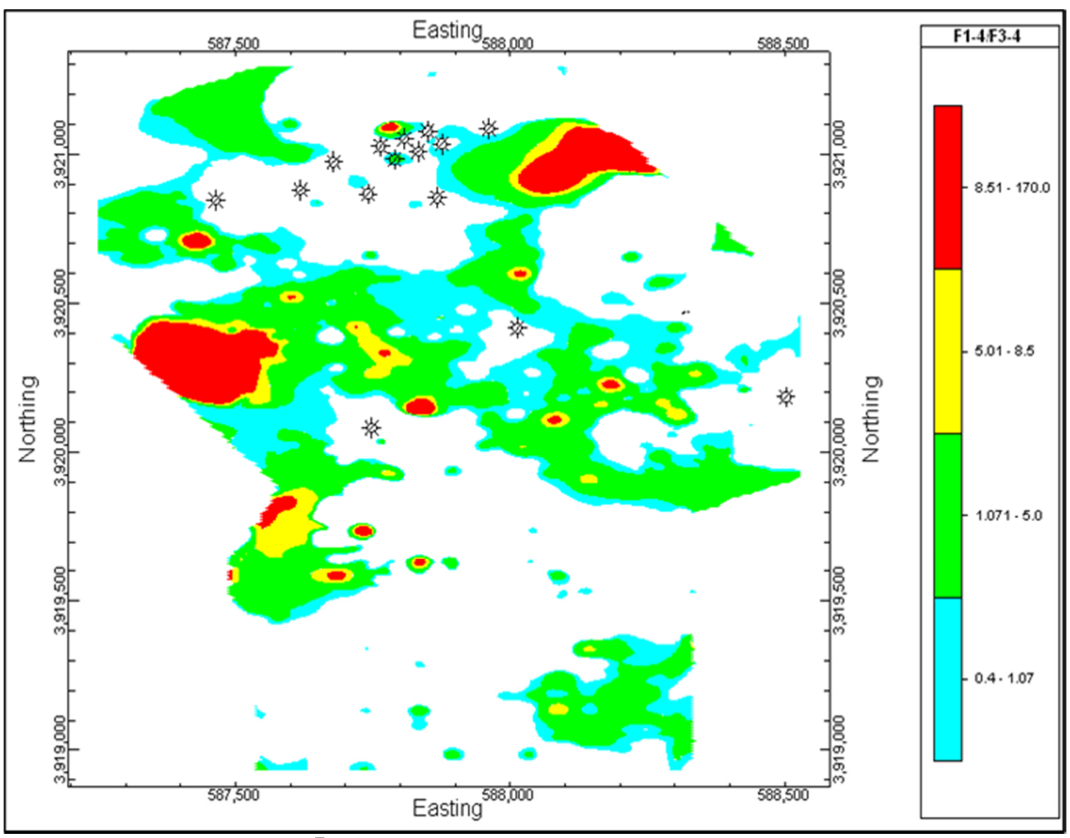


Figure 9. Correlation map for $\frac{F1-4}{F3-4}$ based on the fractal modeling and mineralized rock samples with Au≥100 ppb

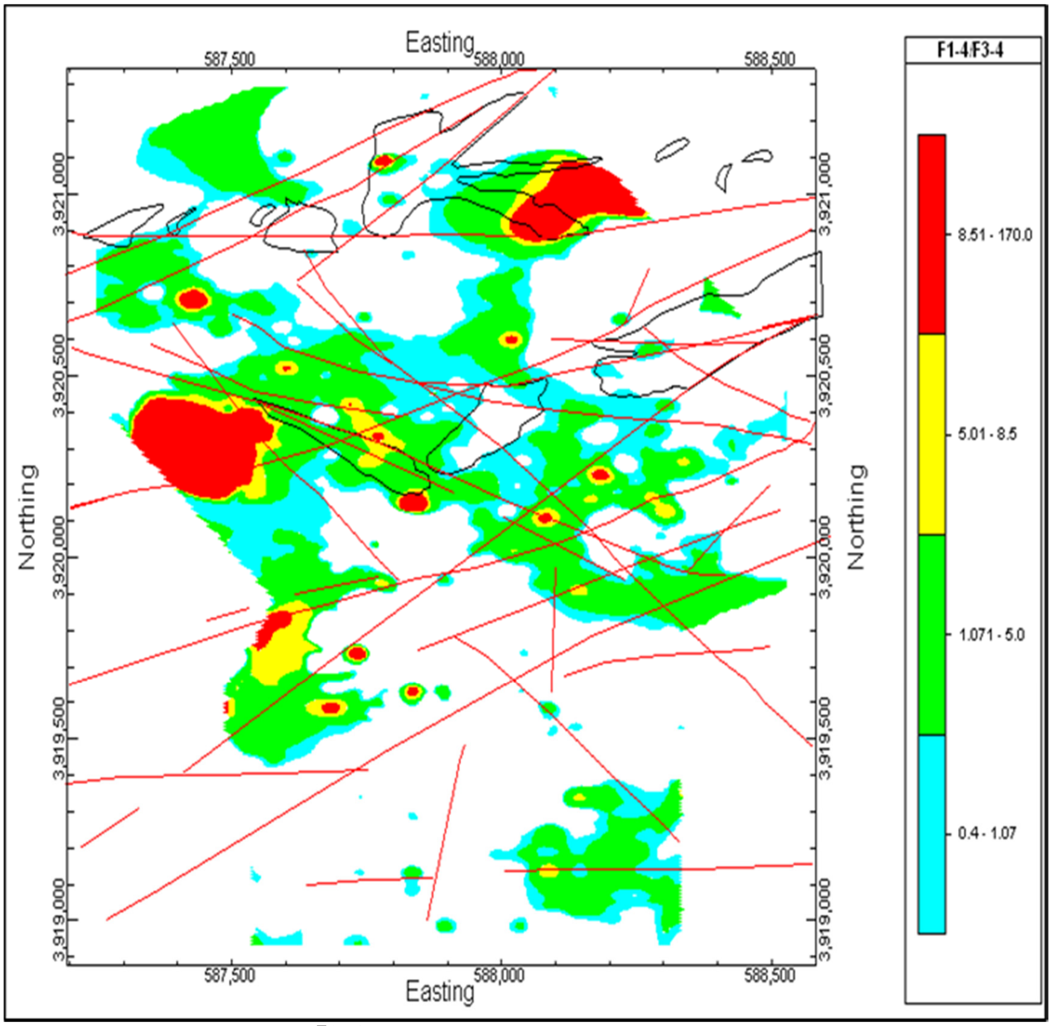


Figure 10. Correlation map for $\frac{F1-4}{F3-4}$ based on the fractal modeling and alteration zones (polygons) and faults (red lines)

Conclusion

This proposed zonality index based on the ratio of factors related to ores and path-finders has proper results in epithermal gold mineralization at the Bardaskan area. The anomalous parts are correlated with geological characteristics (alteration zones and faults) and mineralized samples. Furthermore, these are associated with the second stage of mineralization in the north, especially NE part of this area. Based on these results, the combination between the SFA and fractal modeling can be used for improvement of zonality index. This study indicates that the main prospects for the study area are associated with the main population of the improved zonality index, especially in the northern and NE parts of the studied area. This method is suggested to be applied to other ore deposit types. Subsequently, this methodology is a suitable technique to design grid drilling for detailed exploration.

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