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RESEARCH PAPER

The estimation of Hamedan limestone brittleness index using point load index and porosity test

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Abstract

Brittleness is an important parameter that controls the mechanical behavior and fractures characteristics of rocks in drilling and rock bursting. Lack of accurate evaluation assessing rock brittleness can lead to many risks related to rock mechanics. In this paper, the relationship between brittleness with the ratio of point load index and porosity (PMP), was investigated for Hamedan limestone. Besides, the existing estimation methods for the brittleness index of the rock have been summarized and their application was briefly discussed. In order to estimate brittleness indices and the ratio of point load index to porosity, 18 blocks of Abshineh and Sarab Gyan limestone rocks have been chosen. Abshineh limestone rocks with the Oligo-Miocene age and Sarab Gyan limestones are extracted from Cretaceous rocks. In the following, at first, thin sections of limestone were investigated. Then the physical (porosity (n)) and mechanical properties of limestones were determined. The samples were exposed to point load index (Is_{50}), uniaxial compressive strength (UCS), and Brazilian tensile strength (BTS) tests, and the ratio of their point load to porosity (PMP) was calculated. Then, the relationship between brittleness indices and PMP (univariate regression) was checked. Also, the relationship between brittleness indices was determined with two-variable regression (input variables including porosity and point load index). Finally, the results of different types of relationships were compared. The results illustrate that using the PMP parameter, to predict the values of brittleness indices, obtains more reliable results compared to two-variable regression (n and Is_{50}). Also, the results of experiments showed that the highest agreement between brittleness parameters B3 and B4 with PMP parameter and the coefficient of determination (r^2) are 0.89 and 0.90, respectively.

Keywords: Abshineh, Limestone, Brittleness indices, Point load index, Porosity.

Introduction

Brittleness is one of the mechanical properties of rock and it has a dependency on strength, which indicates the strength of the rock against deformation in the elastic range (Altindag, 2010; Khorasani et al., 2019). Brittleness is an important mechanical property of intact rock because it has a strong influence on cuttability and drillability of rocks, advancement of mechanical excavators, rock burst, stability of underground, and rock behavior under loading process (Singh, 1986; Yagiz, 2008; Altindag, 2002; Gong and Zhao, 2007; Yagiz and Gokceoglu, 2010; Akinbinu, 2016; Yagiz, 2017). The brittleness index is an important index for geomechanical analysis of the reservoir in the stability evaluation of the well and the drilling capability and mechanical properties of the bed of a reservoir. Brittleness is an important and the main feature in rocks that should be considered in all drilling and tunneling projects. In addition, the

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brittleness index is considered as an important feature in various civil and mineral applications. Also, brittleness can control the mechanical properties of rocks (Hussain et al., 2019). In fact, brittleness is one of the most important mechanical properties of rock that has a significant impact on the process of fractures and the response of the rock mass to mining activities, drilling of tunnels, and underground spaces. If the brittleness of the rock increases, the ability to extract decorative rocks with cutting wire of diamond and drilling tunnels decreases by TBM machine. Therefore, a primary evaluation of the speed of extraction of rocks and tunnel excavation can be obtained by determining the brittleness. Researchers have offered different experimental relationships for measuring and determining the brittleness of rocks, but these methods have different results (Yagiz, 2006; Altindag and Guney, 2010; Yarali and Soyer, 2011; Ghobadi et al., 2018; Heidari et al., 2014). However, most researchers consider that brittleness depends on uniaxial compressive strength and Brazilian tensile strength (Altindag, 2002; Kahraman, 2002). Since uniaxial compressive strength and Brazilian tensile strength tests are time-consuming and require standard-sized cores, therefore, several researchers used physical properties (e.g. porosity, density, and water absorption (Heidari et al., 2014; Ghobadi & Naseri, 2016; Ghobadi al., 2018)) and mechanical properties (e.g., Schmidt hammer and point load index (Heidari et al., 2014; Ghobadi & Naseri, 2016; Ghobadi et al., 2018; Yarali & Soyer, 2011; Altindag and Guney, 2010) to evaluate brittleness. The results of these researchers showed that an initial assessment of the brittleness of rocks can be provided by using physical and mechanical properties. Laboratory studies and experiments of TBM machines have shown that the mechanism of cracking and crushing is faster in more brittle rocks; therefore, in order to determine the brittleness of rocks in rock mechanics projects, it is necessary to define an index. For this purpose, various researches have been done by different researchers, some of which are mentioned. Lashkaripour et al., (2018) reported experimental relationships to estimate the brittleness indices of Asmari limestone. The presented equations are highly accurate to predict brittleness based on the uniaxial compressive strength and tensile strength of Brazil. In addition, the results showed that the estimation of brittleness through the compression wave based on the concept of B3 (ratio of uniaxial compressive strength to tensile strength divided by 2) is more accurate than the concepts of B2 (ratio of uniaxial compressive strength minus tensile strength to uniaxial compressive strength plus tensile strength)) and B1 (ratio of compressive strength to tensile strength). There are studies in order to predict the brittleness of the rock using non-destructive methods for tunneling in hard rocks (Kaunda and Asbury, 2016). In this study, the parameters of elastic properties and the wave velocity were used to estimate the brittleness index of the rock by the artificial neural network (ANN) model. A study has been done by Ghobadi et al., (2018) in which the relationship between brittleness parameters and physical properties and also, the concepts of brittleness were determined using physical and mechanical experiments. Based on the obtained results, the highest adaptation between the water absorption percentage test, Schmidt hammer test, and point load index is with the values of brittleness index BI, BI, and B3, respectively. The strength and deformability properties of the rocks tested reveal a decreasing trend with an increasing degree of saturation. The change in the strength and deformability properties with an increasing degree of saturation is related to porosity and the clay content (Karakul and Ulusay, 2013). A study in order to evaluate the brittleness of the rocks and their application is done (Xia et al., 2019). Heydari et al., (2020) presented statistical relationships between brittleness indices and the toughness model. Their research results show a strong relationship between brittleness index B3 and B4 with rock toughness. Ghobadi et al., (2020) examined the engineering geological properties of peridotites in Harsin City of Kermanshah province, and concluded that there is more adaption between engineering properties in dry conditions compared to saturation conditions. Karami et al., (2021) obtained experimental equations for the brittleness index under saturated conditions based on the values of uniaxial compressive strength, tensile strength, and wave velocity. Ghobadi et al., (2022) expressed a relationship between brittleness indices, the ratio of point load index, and porosity (PMP). Jamshidi et al., (2020) presented a novel physico-mechanical parameter (PMP) for predicting the brittleness indices of sandstones. Determination of brittleness indices using physical and mechanical properties parameters is not sufficient to predict brittleness indices. Besides, the uniaxial compressive strength and tensile strength require the cylindrical cores and related devices, which requires a large time and cost to prepare cores and perform tests. While the determination of the brittleness indices with easier tests such as porosity and point load index (no need for cylindrical cores and complex devices) can be done that the time and money are saved. As a result, the brittleness indices of limestone are predicted using the ratio of point load index, and porosity (PMP) in this study. In addition, the purpose of this study is to determine the parameter PMP for Hamedan limestone. Also, the relationships between brittleness indices using bivariate regression (input variables including porosity and point load index) is investigated.

Location and Geologic Setting of the Study

The study area is part of the Sanandaj-Sirjan zone. This zone is one of the most active construction zones in western Iran in the Mesozoic and Cenozoic periods, which has undergone important phases of metamorphism and magmatism before the beginning of the Cenozoic period. Paleozoic sediments are not exposed in this area (Aghanabati, 2004). The spread of limestone units in Hamedan province is very high and has been studied by various researchers (Khanlari et al., 2012; Heidari et al., 2011; Karimi & Taheri, 2010). The study areas have cold and dry climates and mountainous areas. The existence of wide plains and heights has made these areas the field of strong winds. The study area has long and cold winters with long frosts. In this study, two types of limestone located in Hamedan province have been studied, which are related to Hamedan city (Abshineh-Ab region) and Nahavand city (Sarab-Gyan-Gi region) (Figure 1). Abshineh limestone rocks from the Oligo-Miocene age have many fossils and Sarab Gyan limestones are extracted from Cretaceous rocks. Sarab Gyan area consists of massive to thickly layered limestones, which are ovality with a Rubble of fossil shells and some are dark and dolomitic limestone (Khanlari et al., 2015).

Material and method

In this study, in order to determine the subject and method of research, the library method was used. 18 blocks of Abshineh and Sarab Gyan limestone rocks were collected.

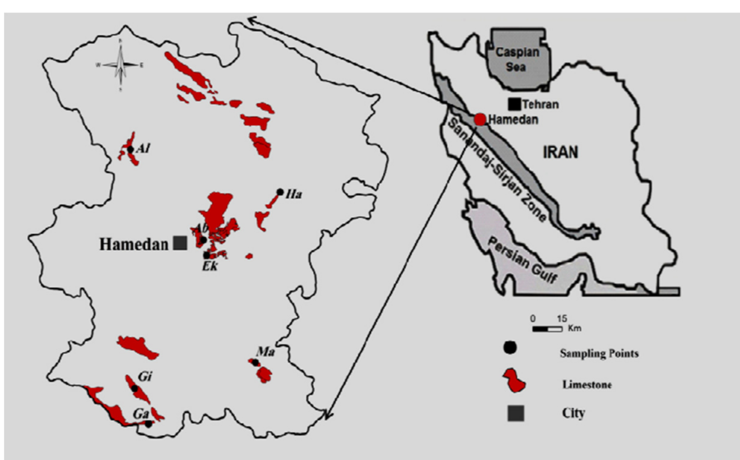

Figure 1. Location of the study area; Ab: Abshineh and Gi: Sarab Gyan (Khanlari et al., 2015)

Uniaxial compressive strength tests (UCS), Brazilian tensile strength (BTS)), The point load index (Is_{50}), and porosity (n) (in accordance with (ISRM, 1981; ISRM, 2007 and ASTM, 2001)) standards were performed. In the following, brittleness indices and physical-mechanical parameter (PMP) were determined using experimental methods. Mechanical properties including uniaxial compressive strength (UCS) and Brazilian tensile strength (BTS) were used to determine brittleness indices (B1, B2, B3, B4, and BI). The PMP parameter was defined using the ratio of point load index (Is_{50}) to porosity (n). Furthermore, the relationship between brittleness indices and PMP parameters was investigated. The relationship between brittleness indices with point load index (Is_{50}) and porosity (n) was checked using two variable regression relationships. At each stage, the validation of the presented relationships was verified using modern methods. Finally, the presented relationships were compared and valid relationships were identified. Also, the results of this study were compared with the results of previous studies and were discussed. The flowchart of the research method is shown in Figure 2.

In Figure (3) showns the frequency histogram, Box Plot, and normal probability diagrams for uniaxial compressive strength (UCS), Brazilian tensile strength (BTS), point load index (Is_{50}), and porosity (n). Based on this figure, the probability distribution of the other properties under study is almost normal except the Brazilian tensile strength. The median of BTS is close to the first quarter, therefore, the BTS has a slight positive skew. The median of the other parameter is approximately in the middle of the box and shows the normal distribution of these parameters. The UCS variation interval is between (30-120 MPa), Is_{50} between (2.5-9.5 MPa), BTS between (4-22 MPa) and porosity between (0-3% and 7-9%). The average values of UCS, Is_{50}, BTS, and porosity are (72.78 MPa), (5.9 MPa), (10.43 MPa), and (3.63%), respectively.
Table (1) shows a summary of the results of the experiments performed. Based on this table, the high values of porosity are related to the Abshineh region and also, the high values of Is_{50} are related to Sarab Gyan region. The reason is due to the age and lithological characteristics of these two types of limestone. The Abshineh limestone rocks from the Oligo-Miocene age have many fossils and Sarab Gyan limestones are extracted from Cretaceous rocks. Sarab Gyan area consists of massive to thickly layered limestones, which are ovality with a Rubble of fossil shells and some are dark and dolomitic limestone.

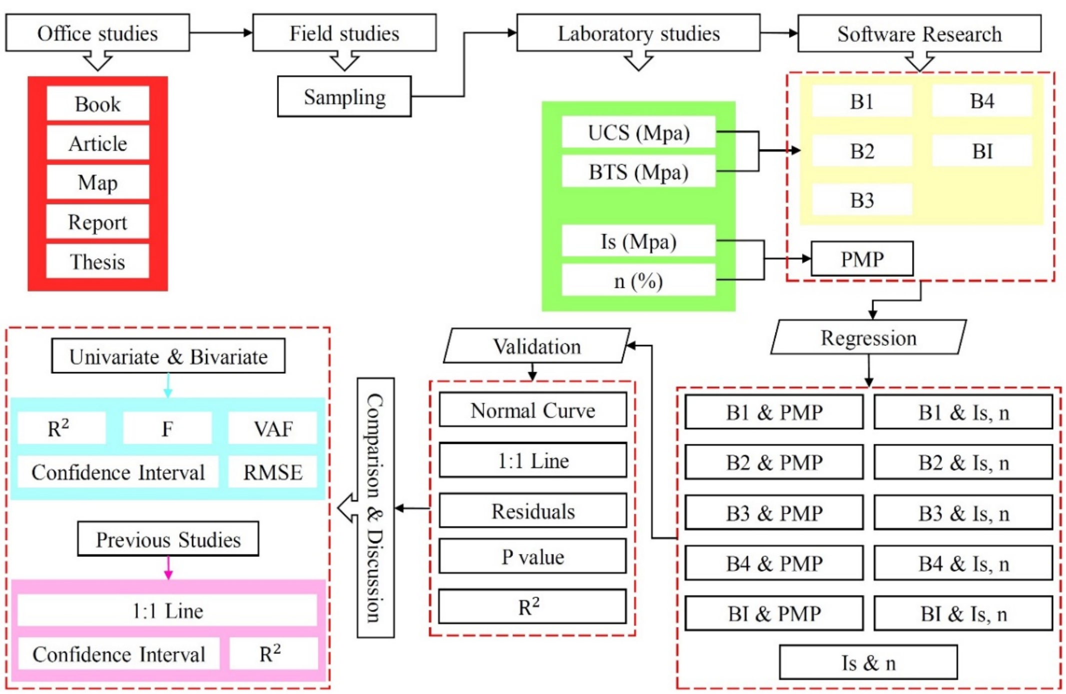

Figure 2. Flowchart of research method

Based on the classification of Broch and Franklin (1970), the strength of these rocks is variated from high to very high and according to the ISRM standard (1981), the strength of these rocks is variated from very low to high. According to Anon's (1979) classification, the porosity of these rocks are classified as in very low to moderate. A view of the rock-cutting device, oven and a number of prepared samples for point load test are demonstrated in Figure 4.

Table 1. The results of the laboratory tests

	Location	Valid N	Mean	Min	Max	Std. Dev.
n (%)	Abshineh	12	5.04	0.98	8.7	3.54
	Sarab-Gyan	6	0.81	0.63	0.97	0.13
Is_{50} (MPa)	Abshineh	12	4.87	2.89	6.66	1.31
	Sarab-Gyan	6	7.93	7.09	9.01	0.63
BTS (MPa)	Abshineh	12	7.82	4.5	16.17	3.17
	Sarab-Gyan	6	15.66	12.59	21.19	3.14
UCS (MPa)	Abshineh	12	61.72	36.9	87	16.01
	Sarab-Gyan	6	94.9	79.25	111.31	12.85

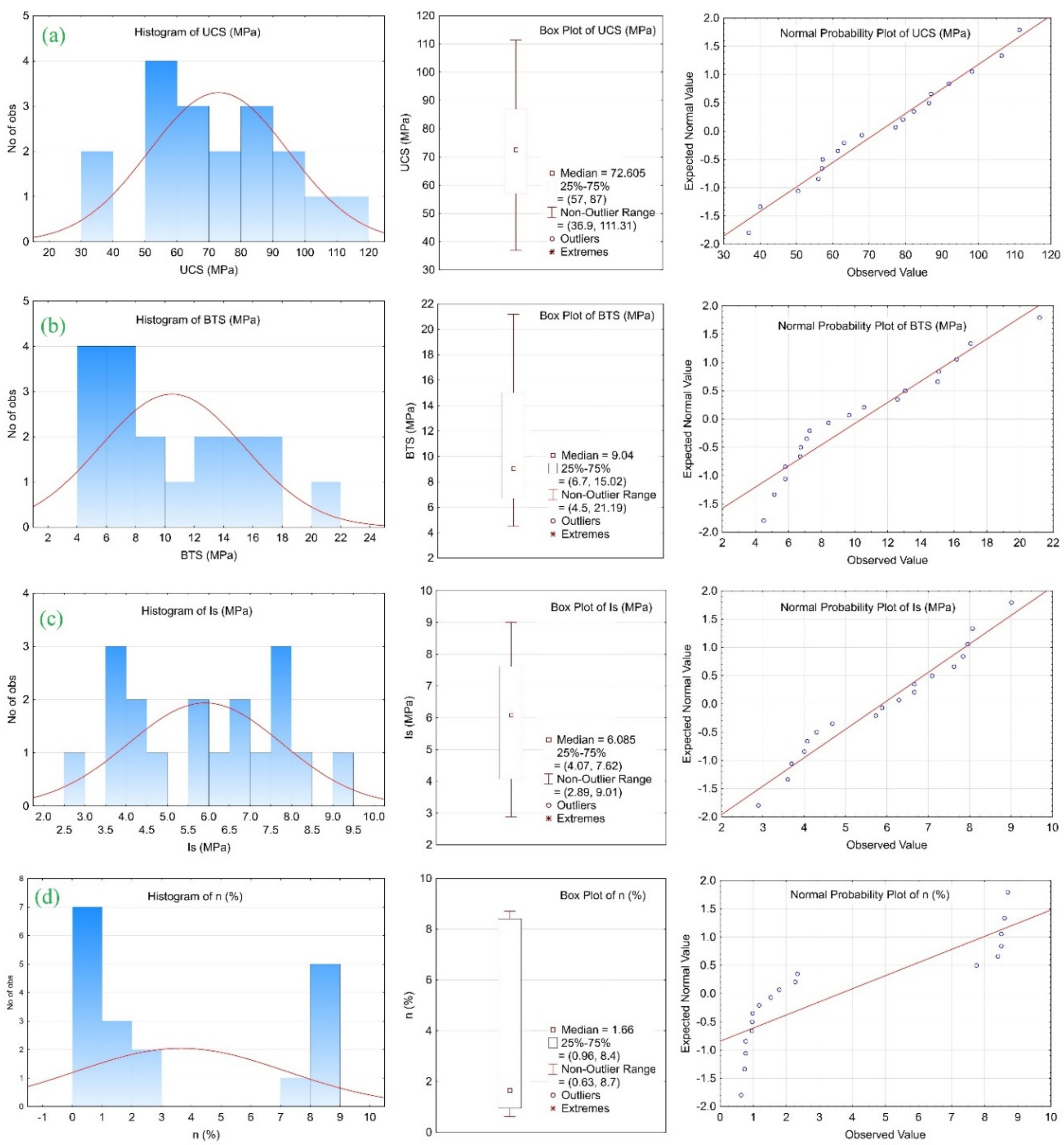

Figure 3. Frequency Histogram, Box Plot, and Normal Probability Chart: (a) UCS, (b) BTS, (c) Is50, and (d) n

Figure 4. (a) the rock-cutting device, (b) A number of prepared samples for point load test (c) oven

Petrology

The study of mineralogy and texture of the studied rocks was performed by preparing thin sections. In addition, the samples were named based on the classifications of Dunham (1962) and Folk (1959) (Khanlari et al., 2015) As mentioned before, the samples of Abshineh have a lot of fossils. Due to the presence of fossils, these samples have a weaker texture than the Sarab Gyan sample. In addition to the looseness of this texture, the removal of fossils over time causes more porosity in these specimens (Figures 5 (a) and (b)). Limestone samples of Sarab Gyan contain a few dolomites. The Abshineh limestones are composed of skeletons of corals, sponges, globigerinas, gastropods, bivalves, and crinoids. There are also detrital components such as quartz and orthoses. The name of the rock is Dunham Biosparite and Folk Bandstone Coral. Also, in Sarab Gyan limestones, microcrystals with small amounts of dolomite and mica were found. The recrystallization of the micrites and their transformation into false coarse sparse crystals is clearly seen (Figures 5 (b)). The name of the rock is calcareous by Dunham Mudstone method and shear by Folk Micrite method.

Brittleness indicators

Brittleness can be considered as small amounts of particle elongation, and fracture separation ratio of the highest compressive to tensile strength (Kahraman and Altindag, 2004). Four different ratios were used to determine the brittleness index in the samples. Table (2) shows the Brittleness concepts used in this research.

In equation 5, the parameter σc is uniaxial compressive strength (in kilopascals), and K is dependent on the type of rock which takes the values from 0.170 to 0.659 (Goktan and Yilmaz, 2005; Kahraman and Altindag, 2004). The value of K is calculated from Table (3), which its value is equal to 0.170 for group rocks such as limestone.

The value of the BI parameter has a wide range which shows the quality of brittle. Table (4) shows the classification values of this parameter.

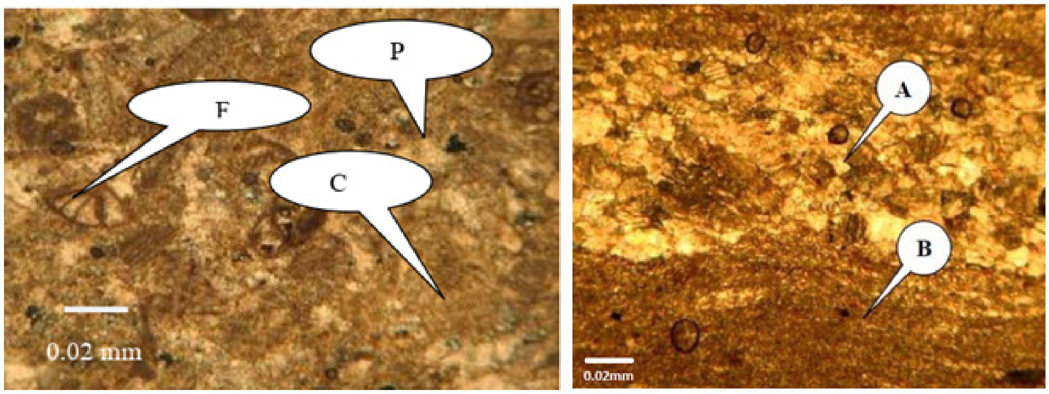

Figure 5. (a): Microscopic image in xpl light of Abshineh limestone consisting of clear calcite and aspartite crystals (C) and porous porosity (P) and Globigerina fossil (F); (b): Microscopic image in xpl light of Sarab Gyan limestone consisting of clear calcite and aspartite crystals (A) and algal cover of micrite (B)

Table 2. The Brittleness concepts used in this research

N	Equation	Source
(1)	$B_1 = \frac{\sigma_c}{\sigma_t}$	(Hunca & Das, 1974)
(2)	$B_2 = \frac{\sigma_C - \sigma_t}{\sigma_c + \sigma_t}$	
(3)	$B_3 = \frac{\sigma_C \times \sigma_t}{2}$	(Altindag, 2002)
(4)	$B_4 = (\sigma_c \times \sigma_t)^{0.72}$	(Yarali and Soyer,2011)
(5)	$BI = 2.065 + K (\log \sigma c)^2$	(Goktan and Yilmaz, 2005)

B_1 ،B_2 ،B_3 و B_4: Brittleness indices

σ_c: Uniaxial compressive strength

σ_t: Brazilian tensile strength

Table 3. Classification of rocks based on K coefficient (Goktan and Yilmaz, 2005)

Rock group	Material type	K
Group a	Carbonate materials with well-developed crystal cleavage (e.g., limestone, dolomite and marble)	0.17
Group b	Lithified argillaceous materials (e.g., mudstone, shale, clay)	0.231
Group c	Arenaceous materials with strong crystals and poorly developed crystal cleavage (e.g., sandstone and quartzite)	0.270
Group d	Fine-grained polymineralic igneous crystalline materials (e.g., andesite, dolerite, diabase and rhyolite)	0.276
Group e	Coarse-grained polymineralic igneous and metamorphic materials (e.g., granite, gabbro, gneiss)	0.659

Table 4. Classification of rocks based on brittleness index (Aftes, 2003)

Class	BI	Description
1	BI>25	Very brittle
2	15<BI<25	brittle
3	10<BI<15	Moderately brittle
4	BI<10	Low brittleness

Figure (6) shows the frequency histogram and Box Plot brittleness indices calculated using the resistance characteristics. As can be seen in this Figure, the frequency distribution curve of all brittleness indices can be considered normal. The range of changes B1, B2, B3, B4, and BI are between (2-18), (0.55-0.9), (100-900 MPa2), (40-220 MPa2), and (2.45-2.8 MPa), respectively.

The average values of B1, B2, B3, B4, and BI are (7.89), (0.75), (411.68 MPa2), (120.33 MPa2) and (2.64 MPa), respectively. The BI parameter was calculated by equation 5 and its value is between 2 and 2.77, which indicates low brittleness according to Table (4). In table (5), the values of brittleness indices are summarized.

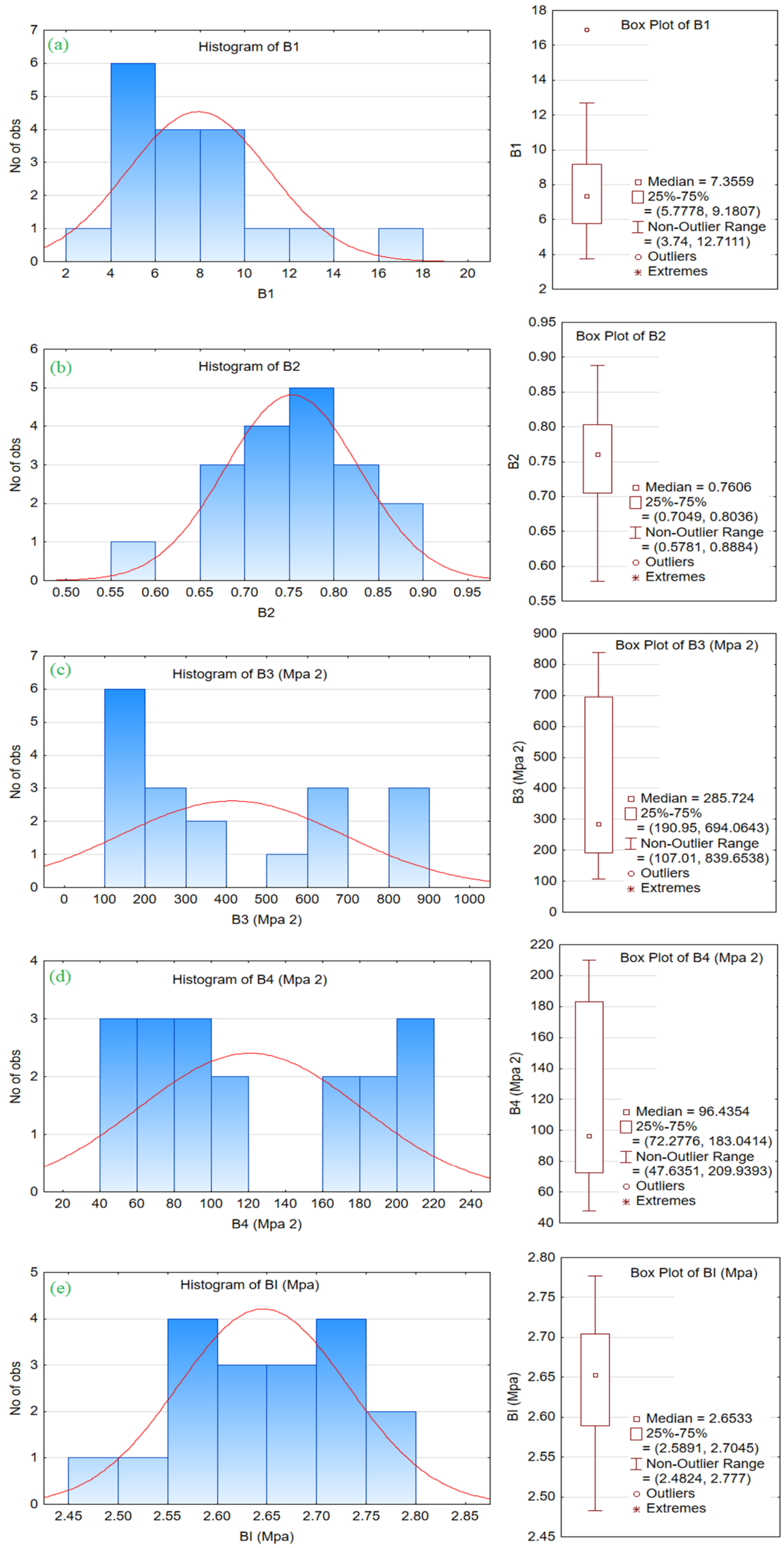

Figure 6. Frequency and Box Plot Histogram: (a) B1, (b) B2, (c) B3, (d) B4, and (e) BI

Table 5. The results of the brittleness index

Brittleness index	Location	Valid N	Mean	Min	Max	Std.Dev.
B1	Abshineh	12	8.67	5.21	16.93	3.5
	Sarab-Gyan	6	6.3	3.74	8.15	1.62
B2	Abshineh	12	0.77	0.68	0.89	0.07
	Sarab-Gyan	6	0.71	0.58	0.78	0.08
B3	Abshineh	12	250.44	107.01	698.54	157.25
	Sarab-Gyan	6	734.16	578.63	839.65	118.72
B4	Abshineh	12	85.41	47.63	183.89	36.08
	Sarab-Gyan	6	190.17	160.57	209.94	22.35
BI	Abshineh	12	2.6	2.48	2.7	0.07
	Sarab-Gyan	6	2.73	2.68	2.78	0.04

Discussion

Relationship between point load index (Is_{50}) and porosity (n)

The results of different studies on different rocks show that the relationship between n and Is_{50} is mainly inverse (Heidari et al., 2014; Khanlari et al., 2015; Jamshidi et al., 2020; Ghobadi et al., 2022). In Figure (7), the changes of n and Is_{50} in different samples are presented. As can be seen in this Figure, the parameter n decreases with increasing Is_{50} and reverse in almost all samples. In the Sarab Gyan region, for all samples (12 to 18), the high values of Is_{50} and very low values of n are observed.

In Figure (8), in order to determine the exact relationship between n and Is_{50}, linear regression (95% Confidence intervals, $\alpha = 0.05$) is verified between these two parameters. The results of regression analysis show that the presented relationship is significant (P-value <0.05) and has an acceptable correlation (R2 = 0.84, F = 80.04) (refer to Table 6).

Also, in order to validate the results of the presented relation, the analysis of the residues of this relation is presented in Figure (9). Residual analysis (difference between real and predicted values) is one of the validation methods of predicted values, has been used widely in recent studies (Lashkaripour et al., 2018; Rahimi Shahid and Hashemian, 2021; Karami et al., 2021). In this analysis, the closer the residual average to zero and the closer the residual distribution to the normal distribution reveal the more reliable the predicted values. As shown in Figure (9), the mean of residuals of the presented regression is approximately zero with a normal probability distribution and according to the normal probability distribution of diagram the residuals are more than 65% of the values on the normal probability line and their distribution can be considered almost normal. In addition, the residues between -0.5 and 05 show the maximum frequency.

Calculate PMP

The results of previous studies showed that there is a direct relationship between brittleness indices with Is_{50} and an inverse relationship with n (Heidari et al., 2014; Ghobadi and Naseri, 2016; Jamshidi et al., 2020; Ghobadi et al., 2022). According to Jamshidi et al., (2020), the PMP equation is defined as equation (6). The PMP parameter is the ratio of point load index to porosity that is used to predict brittleness indices.

$$\text{PMP (MPa)} = \frac{I_{S50}}{n} \quad (6)$$

Where Is_{50} (MPa) and n are the point load index and the porosity, respectively. In Figure (10), frequency diagrams and normal curves, Box Plot, and normal probability diagrams about PMP were calculated. The results showed that the values less than one have the maximum frequency for PMP parameter. The range of PMP changes in this study is between 0.34-12.09 (MPa) and its mean is 4.75 (MPa).

Table 6. Linear regression equation between n and Is_{50}

Equation	R^2	F	P value
$I_{S50} = 7.8 - 0.48\,n$	0.84	80.04	0.00

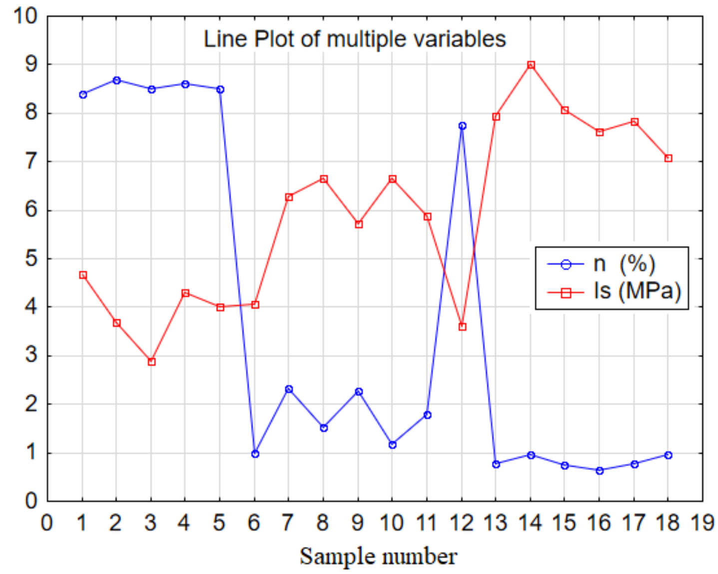

Figure 7. Changes in n and Is_{50} for different samples

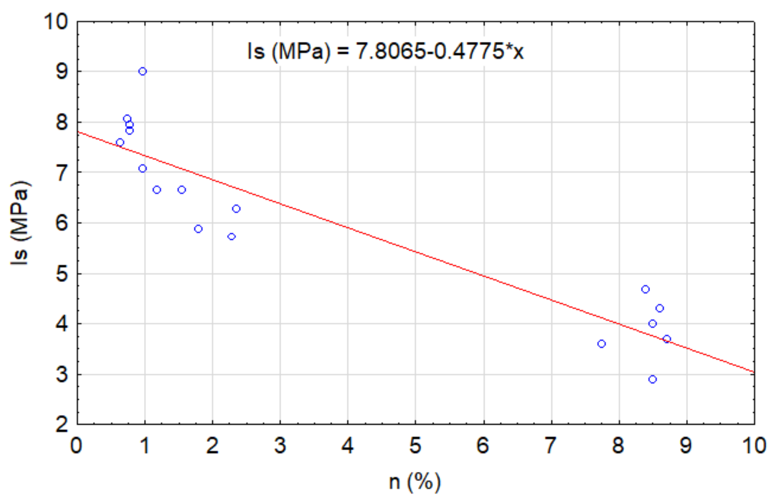

Figure 8. Relationship between n and Is_{50}

According to Figure (10), the distribution of PMP values shows a slight positive skew, since its median is slightly closer to the first quadrant, but with an acceptable approximation, its distribution can be assumed to be normal.

Relationship between brittleness indices and PMP

One of the validations and effective methods in order to determine the engineering properties of rocks, especially the properties that require special samples and more complex devices, is the use regression methods that are still used today. It is widely used in mechanical engineering (Rahimi Shahid, 2015; Chamanzadeh et al., 2016; Rahimi Shahid and Kargaranbafghi, 2021; Rahimi Shahid et al., 2022; Moradi et al., 2021, Rahimi Shahid, 2022). Figure (11) shows the distribution of values of brittleness indices and PMP for different samples.

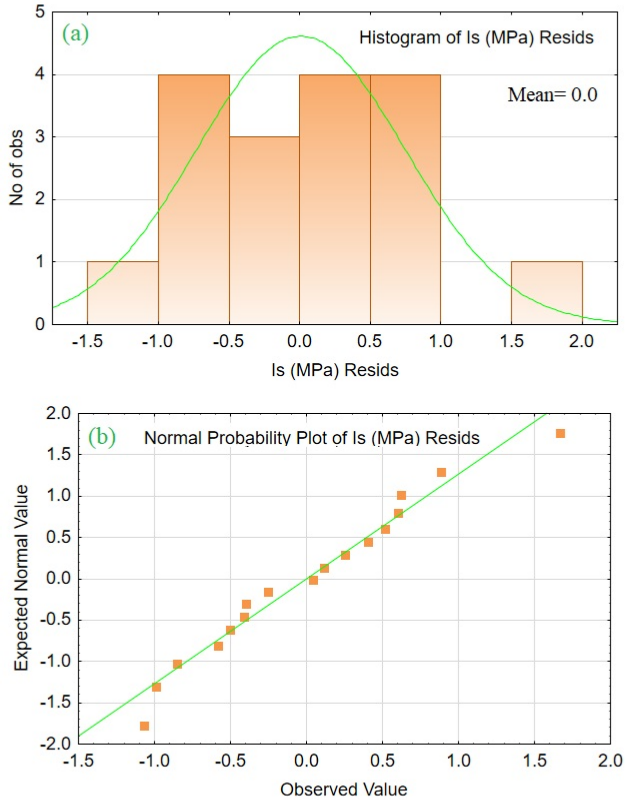

Figure 9. (a) Frequency diagram and normal curve of residuals and (b) Normal probability diagram of regression equation n and Is_{50}

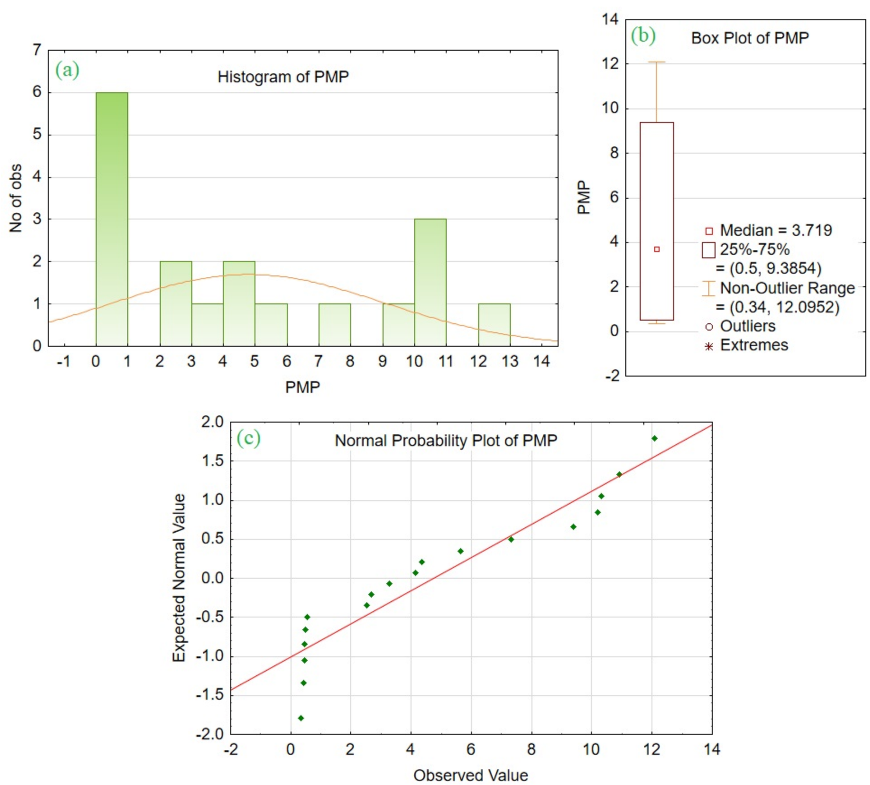

Figure 10. (a) Frequency diagram and normal curve, (b) Box Plot and (c) Normal probability diagram related to PMP

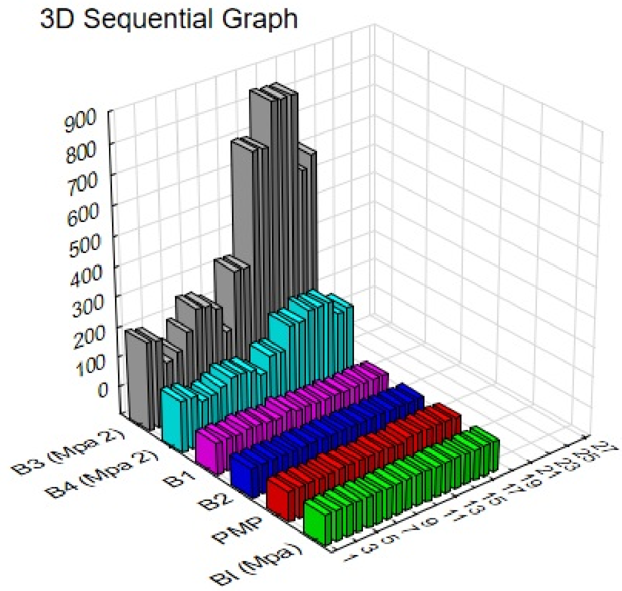

Figure 11. Comparison of the distribution of brittleness index and PMP values for different samples

As can be seen in this Figure, the range of changes in PMP values is closer to the range of changes in BI and B2.

After identifying and removing outlier data, simple regression analysis was used to obtain the rock brittleness relationship using the ratio of point load index (PMP) to porosity in Hamedan limestone. Regression analysis is a simple and practical method. In this method, the relationships between two random variables with a normal distribution are determined. In this study, linear function regression analysis (due to the high accuracy of this type of regression) was used. In this study, the estimated relationships were tested at 95% confidence level. In the regression of linear functions, the coefficient of determination was used. The coefficient of determination is one of the important statistical indicators for evaluating models, which the closer it is to one for a particular model, indicates the high validation of that model. Using the results of a statistic alone cannot show the accuracy and validation of regression relationships. Therefore, after determining the regression relationships, it is necessary to check the accuracy, and significance of these relationships. In this study, the P-value test was used to determine the significance and accuracy of relations. Also, to evaluate the accuracy and validation of relationship results of residual analysis, compare the frequency distribution and normal curve of input values and predicted regression. The simple linear curve fitting of each relationship between brittleness indices and PMP is shown in Figure (12). In addition, the analysis results of each relationship are shown in Table (7).

According to Table (7), the relations B3, B4 and BI at the level of significance are 95% (α = 0.05), significant (P-value <0.05) and the relations B1 and B2 are meaningless (P-value> 0.05). The relationships of B3, B4 and BI with PMP are of the direct type and the maximum accuracy of the relationships based on the value of the coefficient of determination obtained between PMP and B3 and B4 is R2 = 0.89 and R2 = 0.90, respectively (Table 7).

The frequency histogram and the normal probability diagram of the residuals of the relationships B3, B4 and BI with PMP are shown in Figure (13). Residual analysis shows that the residual distribution is almost normal and the average residual tends to zero (Figure 13). Therefore, the residual analysis confirms the high validation of the presented regressions. In fact, the normal distribution of the residuals indicates that the positive and negative errors of the relations coincide and in general the estimation error is small and negligible.

Figure 14 shows the frequency histogram and the normal distribution curve of the real and predicted values of brittleness indices (relationships B3, B4, and BI with PMP). Based on this Figure, the normal distribution curve of the real and predicted values of the brittleness indices are almost identical. According to Figure (14), the relationships B3, B4, and BI with PMP have reliable results for values greater than 200, 60, and 2.60, respectively. Fitting the real and predicted values of brittleness indices (relationships B3, B4, and BI with PMP) also indicates a coefficient of determination (R2) above 0.87 and confirms the high accuracy and validation of the proposed relationships (Figure 15). In fact, this figure shows that the proposed relationships for B3 are less than the 200 predicted values less than the actual values. Also, the predicted values are higher than the actual values for B4 less than 60 and for BI less than 2.6.

Table 7. Linear regression equations between brittleness indices and PMP

Equation	R^2	F	P value
$B1 = 8.12 - 0.16\ PMP$	0.09	1.53	0.23
$B2 = 0.77 - 0.004\ PMP$	0.06	1.08	0.31
$B3 = 88.79 + 61.10\ PMP$	0.89	121.87	0.00
$B4 = 49.63 + 13.37\ PMP$	0.90	134.48	0.00
$BI = 2.6 + 0.01\ PMP$	0.87	78.55	0.00

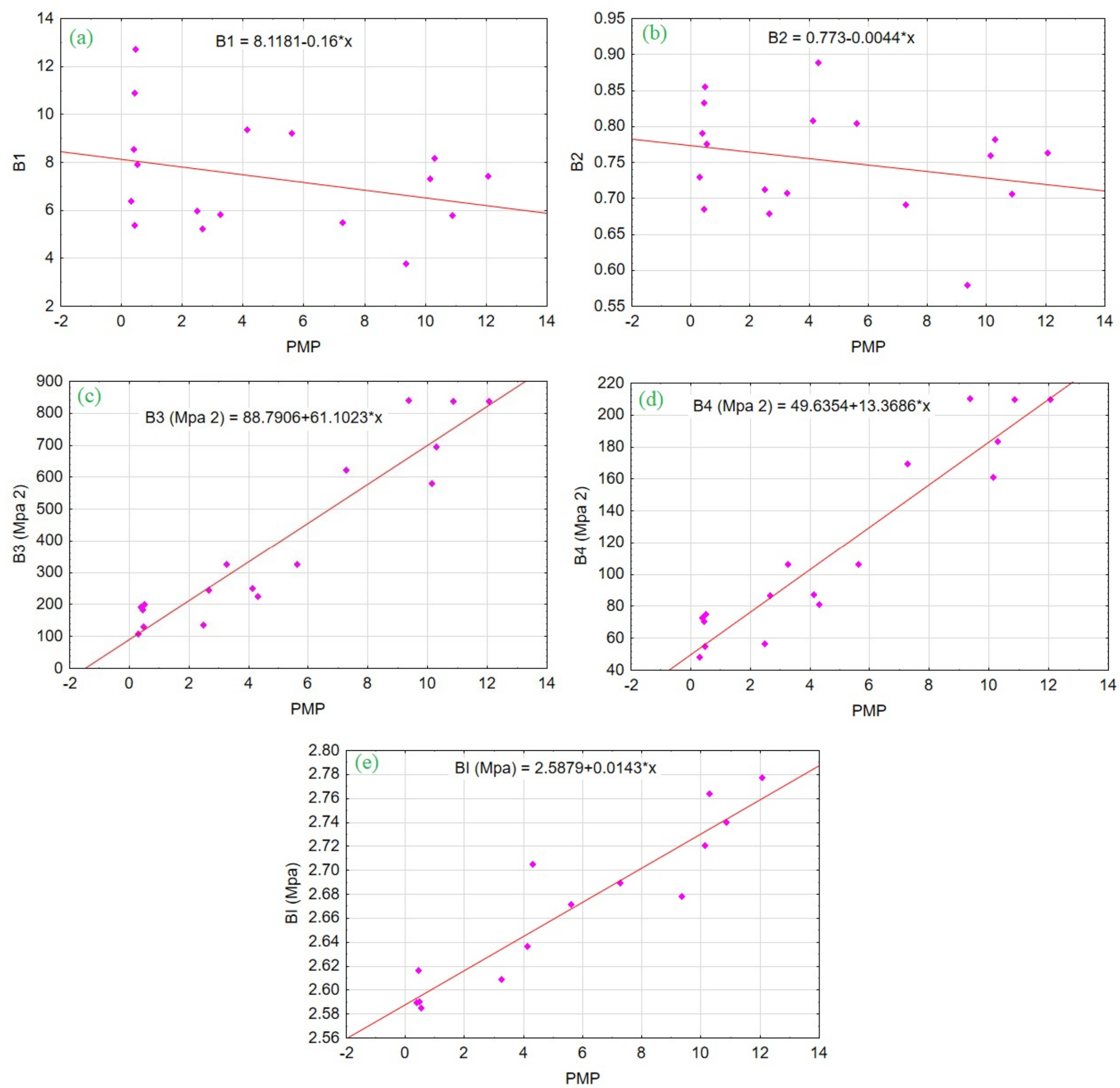

Figure 12. Relationships between brittleness indices and PMP

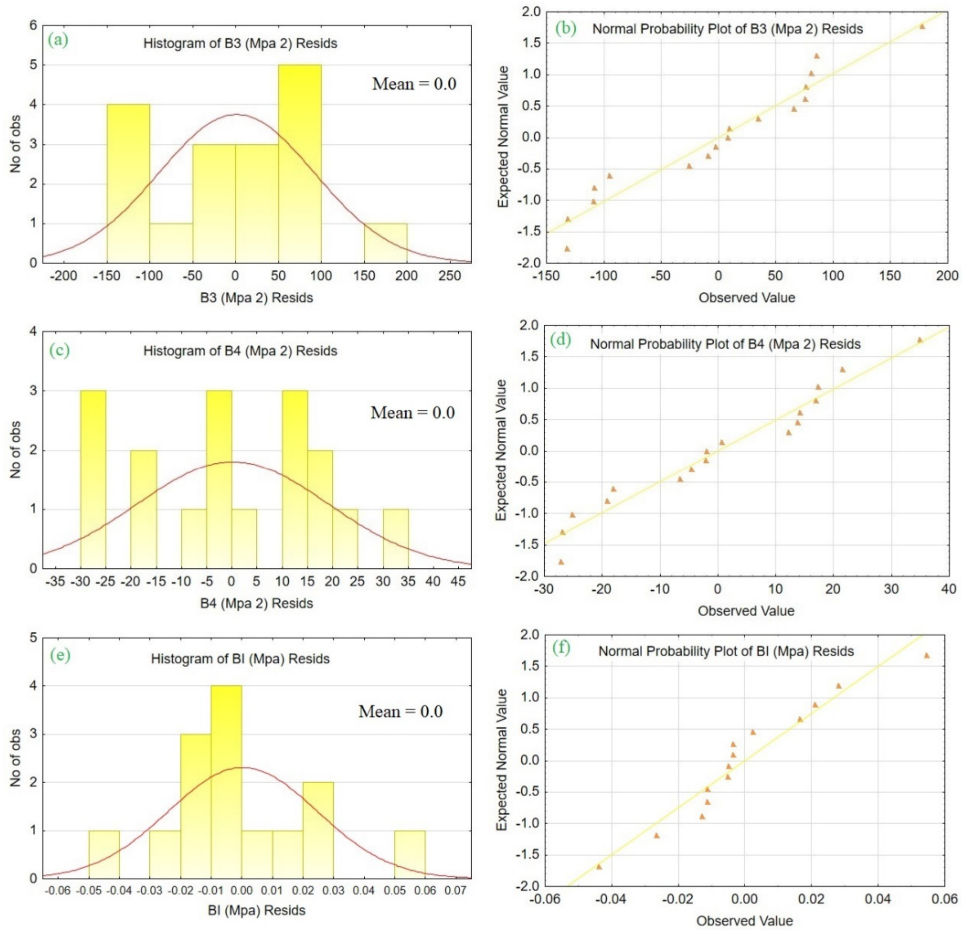

Figure 13. Frequency diagram and normal residual curve and normal probability residual diagram of regression equations of brittleness indices

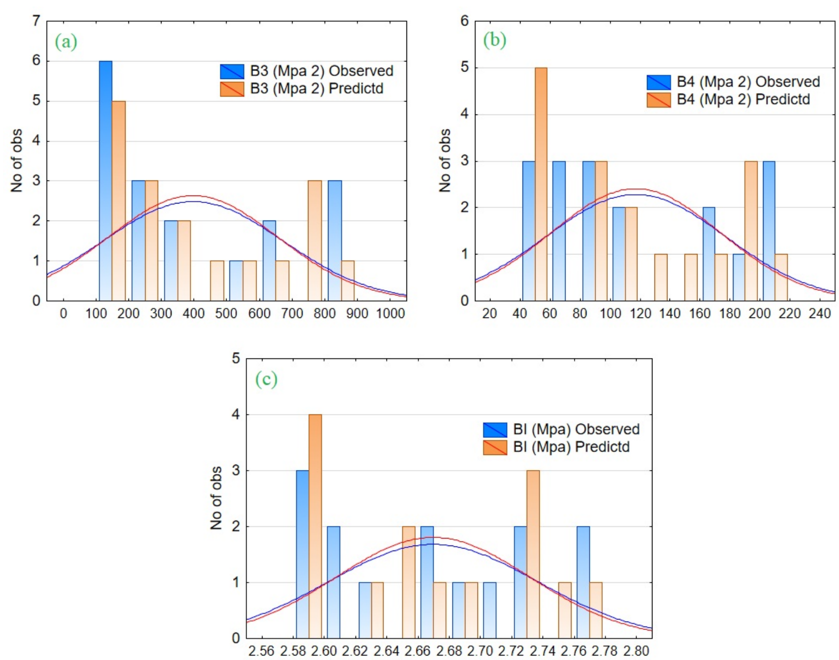

Figure 14. Comparison of frequency distribution and normal curve of input and predicted values of univariate regression equations (a) B3, (b) B4, and (c) BI

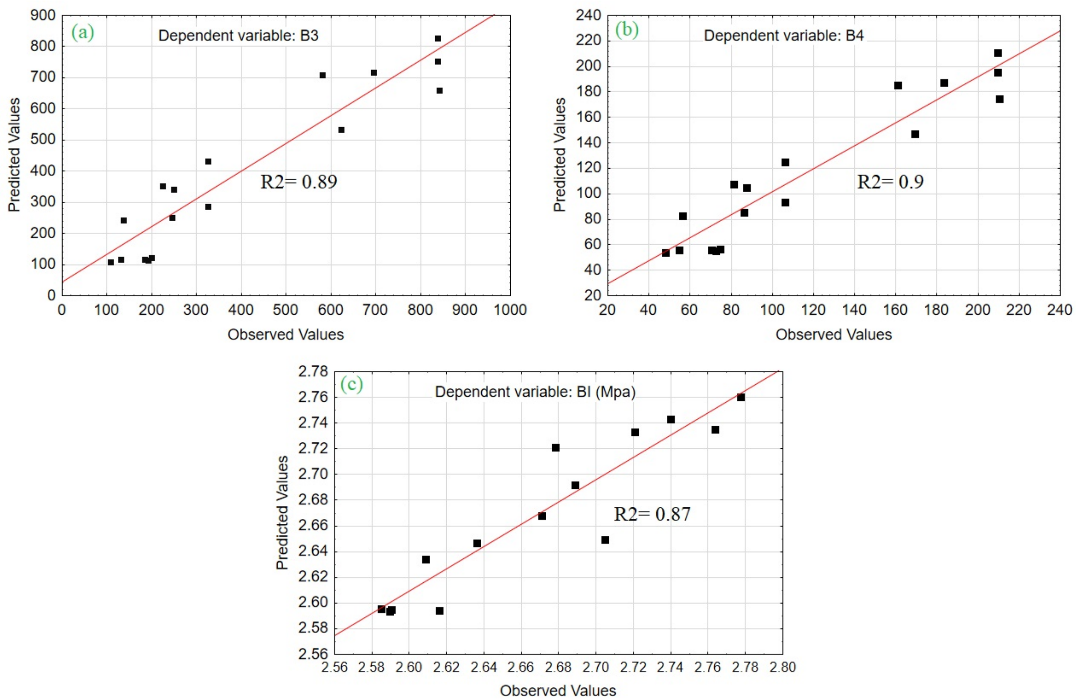

Figure 15. Correlation between input and output values of univariate regression equations of brittleness indices (a) B3, (b) B4, and (c) BI

Calculation of brittleness indices using bivariate regression

In this section, two-variable regression with inputs n and Is_{50} is used to predict brittleness indices. In Figure (16), the three-dimensional distribution of n and Is_{50} versus brittleness indices is shown. It is noteworthy, almost the B1 and B2 parameters are increased by increasing n and decreasing Is_{50}, while for other brittleness indices with increasing n and decreasing Is_{50} the B3, B4, and BI parameters decrease (Figure 16).

In the Table (8), the relationships of two-variable linear regression of n and Is_{50}, brittleness indices (95% confidence intervals, $\alpha = 0.05$), and the analysis are shown. As can be seen in this Table, the relationship between B1 and B2 with n and Is_{50} is not significant (P-value > 0.05) while the relationship between B3, B4, and BI with n and Is_{50} is significant (P-value <0.05)) and have high accuracy (R^2> 0.72 and Mean of Residual = 0). Nevertheless, these relationships show a lower correlation coefficient than univariate regressions.

Comparison of the results of univariate and multivariate regressions

After predicting the brittleness indices using univariate regression (With PMP) and bivariate regression (With Is_{50} and n), the results of these regression equations (significant relationships) were compared. For this purpose, four different performance evaluation indices including correlation coefficient, F statistic, root mean square error (RMSE), and VAF value were used (Equations 7 and 8). The correlation coefficient is one of the important statistical indicators for evaluating and validating models which the closer it to 1 for a particular model, indicates the higher the accuracy of the model. In statistical analysis, the higher the F statistic, indicates the higher the validation of the model. On the other hand, the closer the value of the mean square root of the error is to zero, indicates the higher the accuracy of the model and its greater compliance with the real values. In addition, the VAF value was used to determine the reliability and error rate of different methods and models. The closer the value of this index is to 100%

for a particular model, indicates the greater accuracy and validation. The root means square error (RMSE) equation and the VAF value are given below.

$$\mathrm{VAF} = [1 - (\mathrm{var}\,(y - y')/\mathrm{var}\,(y))] \times 100 \tag{7}$$

$$\mathrm{RMSE} = \sqrt{\left(\frac{1}{N}\right) \times \sum_{i=1}^{N} (y - y')^2} \tag{8}$$

Table 8. Results of bivariate regression analysis of brittleness indices with n and Is_{50}

Equation	R^2	F	P value	Mean of Residual
$B1 = -0.29\,Is + 0.13\,n + 8.57$	0.18	1.61	0.23	0.00
$B2 = -0.017\,Is - 0.002\,n + 0.86$	0.12	1.02	0.38	0.00
$B3 = 143.21\,Is + 9.3\,n - 499.65$	0.73	19.89	0.00	0.00
$B4 = 30.41\,Is + 1.31\,n - 71.14$	0.75	21.52	0.00	0.00
$BI = 0.02\,Is - 0.006\,n + 2.56$	0.72	14.29	0.00	0.00

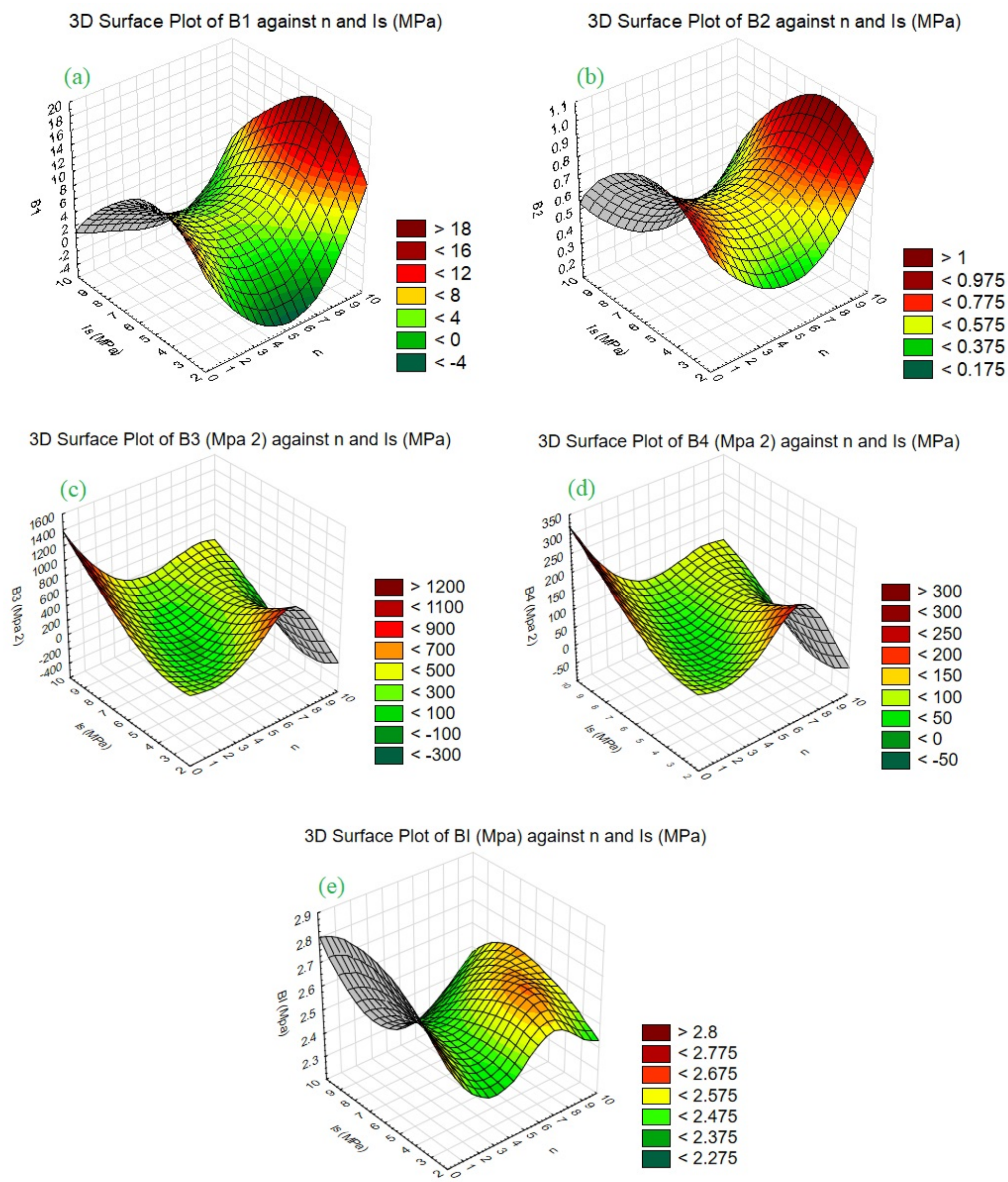

Figure 16. Three-dimensional changes of n and Is_{50} versus (a) B1, (b) B2, (c) B3, (d) B4 and (e) BI

In these equations, y is the measured value, y' is the predicted value, and N is the number of data (Hocking, 1976). The results of this study showed that the determination of brittleness indices B3, B4 and BI is possible using n and Is50 (bivariate regression) and PMP (univariate regression).

In Figure 17, comparisons of R^2, F, % VAF and RMSE values between univariate and bivariate regressions (significant relationships) are performed. As can be seen in this Figure, the values of R^2, F and % VAF are related to the univariate relationships of brittleness indices (B3, B4 and BI) with PMP relative to the relationships between B3, B4 and BI. With n and Is_{50} show higher values. The results show that estimation of brittleness indices using PMP greatly increases some statistics such as F. The value of this statistic in this study is more than 78 for one variable regression and less than 40 for two variable regressions. Also, RMSE values related to the relationships between B3, B4 and BI with n and Is_{50} show higher values. Therefore, the results of this study show that the use of PMP parameter to predict the values of brittleness indices compared to bivariate regression (n and Is_{50}) in limestone provides more reliable results.

In Figure 18, a comparison between univariate and bivariate regressions (significant relationships) is performed using confidence interval graphs (95% Confidence intervals). The maximum and minimum real and predicted values of B3, B4 and BI in univariate regression (With PMP) are closer to each other, while the same values show a much greater difference for bivariate regression (With Is_{50} and n). The mean of the actual and predicted values of B3, B4, and BI in the univariate (With PMP) and bivariate (With Is_{50} and n) regression are approximately equal (Figure 18). Therefore, the comparison results of confidence interval graphs also indicate that the use of PMP parameter to predict the values of brittleness indices than bivariate regression (n and Is_{50}) gives more reliable results.

Comparison of the results of this study with similar studies in the past

Jamshidi et al., (2020) introduced a new parameter called PMP for predicting brittleness indices in sandstone. These researchers estimated the brittleness indices B1, B2, and B3 by this parameter (Table 9).

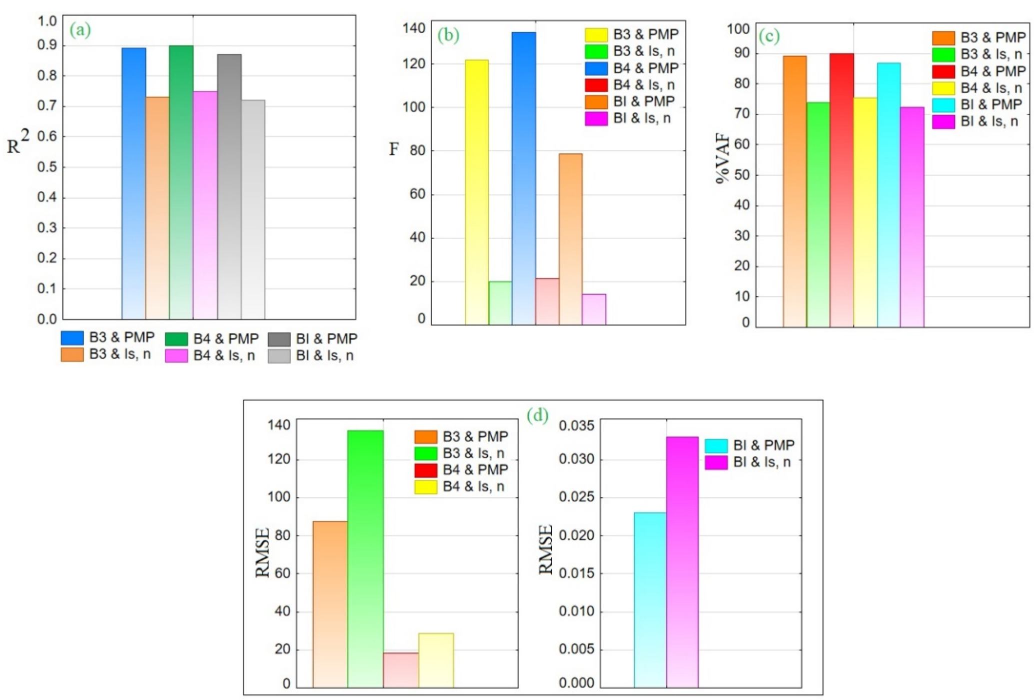

Figure 17. Comparison of (a) R^2, (b) F, (c) % VAF and (d) RMSE univariate and bivariate regressions

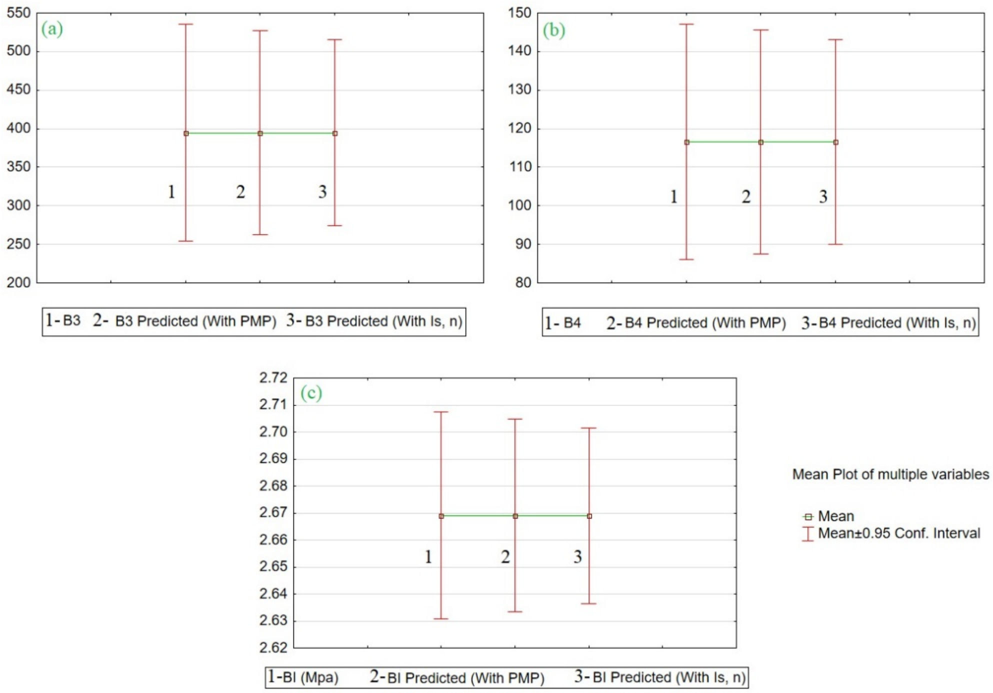

Figure 18. Confidence interval charts (at 95% significance level) for the relationships presented in this study: (a) B3, (b) B4, and (c) BI

Table 9. Comparison of regression equations of brittleness indices in this study with previous studies

Equation	R^2	Rock	Reference
$B1 = 8.12 - 0.16\ PMP$	0.09	Limestone	This study
$B2 = 0.77 - 0.004\ PMP$	0.06		
$B3 = 88.79 + 61.10\ PMP$	0.89		
$B4 = 49.63 + 13.37\ PMP$	0.90		
$BI = 2.6 + 0.01\ PMP$	0.87		
B1=1.4402Ln (PMP)+2.7663	0.95	Sandstone	Jamshidi et al., (2020)
B2=0.0345Ln (PMP)+0.6507	0.94		
B3=95.728Ln (PMP)-201.62	0.93		
$B1_{Dry} = 15.97 - 0.1767\ PMP_{Dry}$	0.78	Sandstone	Ghobadi et al., (2022)
$B2_{Dry} = 0.9621 - 0.004167\ PMP_{Dry}$	0.80		
$B3_{Dry} = 44.11 + 2.835\ PMP_{Dry}$	0.92		
$B4_{Dry} = 30.78 + 0.802\ PMP_{Dry}$	0.92		
$BI_{Dry} = 2.901 - 0.0009\ PMP_{Dry}$	0.23		
$B1_{Sat} = -0.205 + 0.333\ PMP_{Sat}$	0.87		
$B2_{Sat} = 0.6529 + 0.005315\ PMP_{Sat}$	0.88		
$B3_{Sat} = 135.6 - 2.159\ PMP_{Sat}$	0.57		
$B4_{Sat} = 59.03 - 0.79\ PMP_{Sat}$	0.57		
$BI_{Sat} = 2.728 - 0.00002\ PMP_{Sat}$	0.00		

The researchers used nonlinear regression to estimate the brittleness indices. Based on their results, there is a direct and valid relationship between brittleness indices and PMP ($R^2 \geq 0.93$). In another study by Ghobadi et al., (2022), the brittleness indices B1, B2, B3, B4, and BI were predicted using the PMP parameter in both dry and saturated conditions for sandstone. They used linear regression to determine the brittleness indices using PMP. The results of these studies are presented in Table (9). In the present study, the brittleness indices B1, B2, B3, B4, and BI were predicted by the PMP parameter for limestone. The correlation coefficients of the equations in Table (9) are shown in Figure (19). Comparison of the results shows that the determination of brittleness indices B1 and B2 provides reliable results using PMP in sandstones while the determination of brittleness indices B1 and B2 is impossible in limestone. The determination of brittleness indices B4 and BI in limestone provides more accurate results than in sandstone using PMP. The results of this study and the study of Ghobadi et al (2022) show that the estimation of brittleness indices using linear regression provides reliable results for limestone.

In Figure 20, a comparison of the correlation between the input and output values of the univariate regression equations of the brittleness indices of this study with previous studies has been shown. As can be seen in this Figure, the results of the present study are more adaptable to the results of the studies (Ghobadi et al., 2022).

Conclusion

This article presented relationships between brittleness parameters with the ratio of point load index and porosity (PMP) for limestones of Hamedan. Based on this study, the following results can be presented: 1) The correlation is between the ratio of point load index and porosity (PMP) with brittleness parameters (BI, B3, and B4) which their values are 0.89, 0.90, and 0.87, respectively. Also, there is a weak relationship between the brittleness parameters (B1, B2) and PMP.

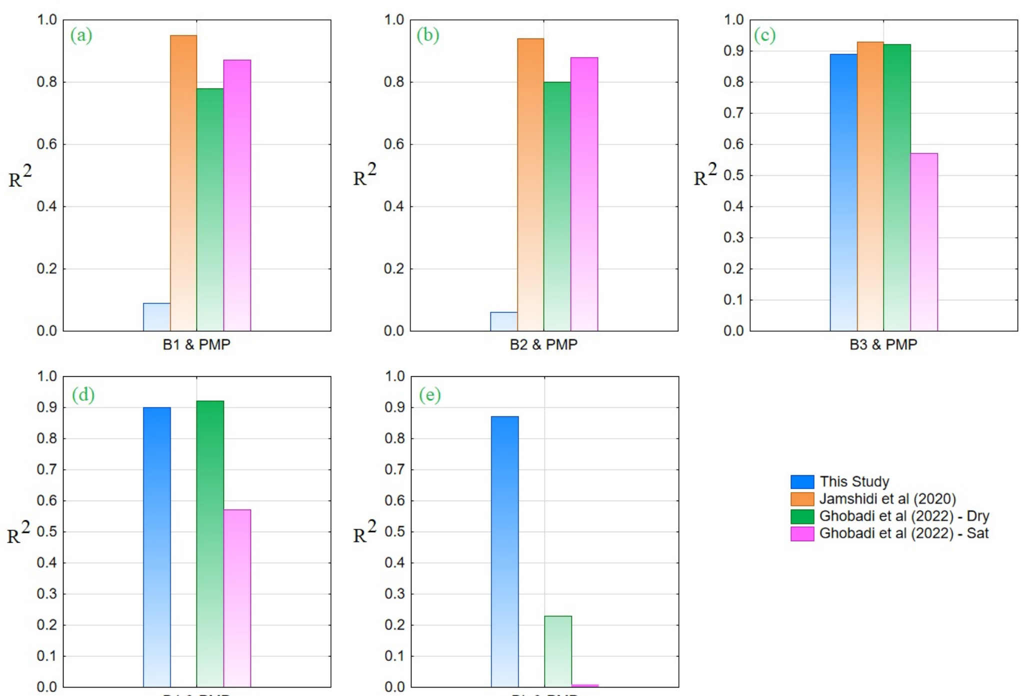

Figure 19. Comparison of correlation coefficient (R^2) of the equations of the present study with similar studies of the past

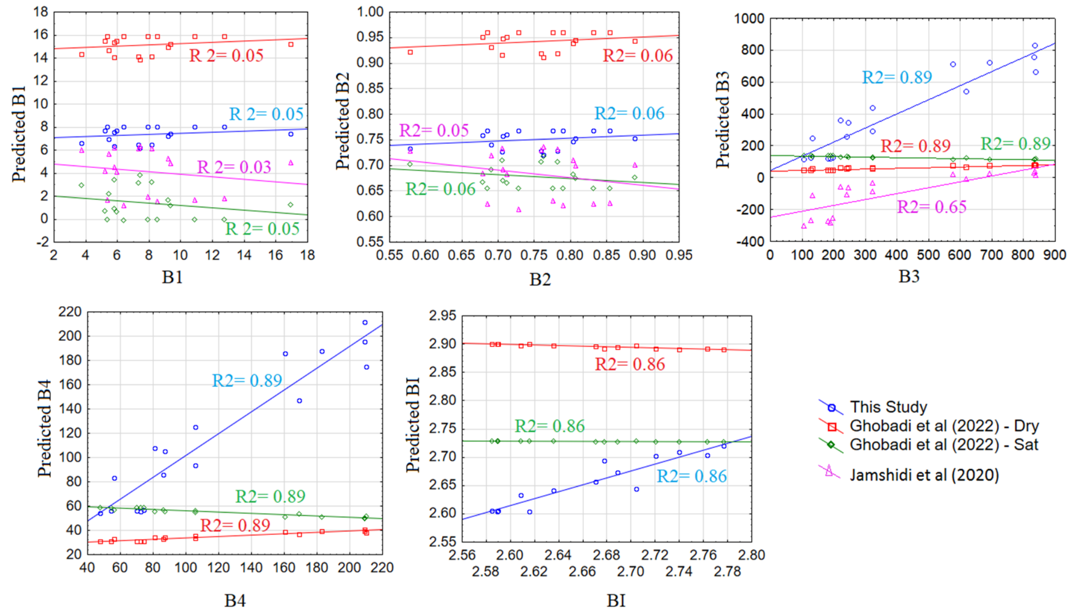

Figure 20. Comparison of correlation between input and output values of univariate regression equations of brittleness indices of this study with similar studies of the past

2) There is a weak relationship between the brittleness parameters B1 and B2 with n, and I_{s50} while significant correlations exist between the brittleness parameters B3, B4, and BI with n, and I_{s50}. 3) In limestone rock, using the PMP parameter provides more reliable results to predict the value of brittleness indices in comparison to two-variable regression n and I_{s50}. 4) Comparison of the results shows that using the PMP parameter presents reliable results to determine brittleness indices B1 and B2 in sandstones while the determination of brittleness indices B1 and B2 using PMP is impossible in limestone. Determination of brittleness indices B4 and BI using PMP provides more accurate results than sandstone in limestone.5) Increasing the number of regions, the diversity of lithology and increasing the number of samples can lead to more acceptable results. Also, the results can be generalized to other limestones of other regions with higher reliability by increasing the diversity in lithology.

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