The effect of estimation methods on fractal modeling for anomalies' detection in the Irankuh area, Central Iran

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Abstract
This study aims to recognize effect of Ordinary Kriging (OK) and Inverse Distance Weighted (IDW) estimation methods for separation of geochemical anomalies based on soil samples using Concentration-Area (C-A) fractal model in Irankuh area, central Iran. Variograms and anisotropic ellipsoid were generated for the Pb and Zn distribution. Thresholds values from the C-A log-log plots based on the estimation methods revealed the presence of various geochemical anomalies within estimation variances which were compared in both methods. The comparison among the estimation variances for different geochemical anomalies based on the C-A fractal model indicated that the estimation variance is less in the OK method especially for extremely and highly Zn-Pb anomalies. The estimated variances for different Zn anomalies via OK and C-A fractal method are lower than fractal modeling obtained by IDW estimation. However, extremely and highly Pb anomalies due to OK method have estimation variances lower than IDW method. Based on the results, main Zn and Pb anomalies are situated in the NW part of the area.

Keywords: *Concentration Area (C-A) Fractal Model, Estimation Variance, Inverse Distance Weighted (Idw), Irankouh, Ordinary Kriging (OK).*

Introduction

Fractal/multifractal modeling has been widely applied in different branches of geosciences and mining engineering, especially in spatial modeling of geochemical anomalies and mineralized zones (e.g., Cheng *et al*., 1994; Agterberg, 1995; Goncalves *et al*., 2001; Afzal *et al*., 2010, 2011, 2012, 2013; Agterberg, 1995; Delavar *et al*., 2012; Heidari *et al*., 2013; Wang *et al*., 2011, 2012; Yasrebi *et al*., 2014; Rahmati *et al*., 2015). The Concentration-Area (C-A) fractal model by Cheng *et al*. (1994) has been used to discriminate between geochemical anomalies and background via thresholds' values (breakpoints). As a result, the C-A fractal model is considered as a proper method to describe spatial distributions of different attributes (e.g., ore elements in this scenario) for the various ore deposits (Goncalves *et al*., 2001; Shamseddin Meigoony *et al*., 2014; Aramesh Asl *et al*., 2015).

Geostatistical methods are utilized for estimation and interpolation of different regional variables in 1D, 2D, or 3D illustrations. Employment of an accurate estimation method with respect to geometry and geological properties of different ore deposits and also sampling patterns is a problematic issue in reserve estimation (David, 1970; Journel, 1983; Shahbeik *et al*., 2014).

Linear and non-linear Kriging methods, Inverse Distance Weighted (IDW), and interpolating polynomials and splines have been used for resource estimation. Based on collected data and grid sampling, the proper estimator were chosen (Matheron, 1967; Marechal & Serra, 1971; Laslett *et al*., 1987; Laslett and McBratney, 1990; Weber and Englund, 1992, 1994; Gallichand and Marcotte, 1993; Laslett, 1994; Brus *et al*., 1996; Declercq, 1996; Phillips *et al*., 1997; Zimmerman *et al*., 1999). Moreover, there are several studies which have compared Ordinary Kriging (OK) and IDW (Zimmerman *et al*., 1999; Shahbeik *et al*., 2014).

Estimation of ore element distribution is an important aspect of mineral exploration (Moon *et al*., 2005; Heidari *et al*., 2013). Determination of an interpolation method is important for decreasing variance estimation and increasing accuracy for 2D and 3D evaluation (Dimitrakopoulos *et al*., 2007; Juan *et al*., 2011; Parhizkar *et al*., 2011). In addition, selection of an estimation method is essential for using fractal modeling, especially in the C-A model (Cheng *et al*., 1994; Yasrebi *et al*., 2013; Soltani *et al*., 2014). Furthermore, accuracy of estimation method and its corresponding variance affect the C-A fractal/ multifractal model.

The main purpose of this paper is to separate and compare estimation variances for various Pb and Zn anomalies which were derived by the C-A fractal model based on IDW and OK estimation methods in the Irankuh area, Central Iran. Moreover, classical statistical parameters were described for Pb and Zn.

Material and method

The Concentration-Area (C-A) fractal model by Cheng *et al.* (1994) is utilized to outline the anomalies and background in order to describe the distribution of ore element concentrations. This model is expressed in the following form:

$$A(\rho \leq \upsilon) \propto \rho^{-a1}; \quad A(\rho \geq \upsilon) \propto \rho^{-a2} \qquad (1)$$

where $A(\rho \leq \upsilon)$ and $A(\rho \geq \upsilon)$ show areas (A) with concentration values (ρ) that are, respectively, smaller and greater than contour values (υ) and a1 and a2 are exponents. Log-log plots of concentration values versus areas, certain concentration contours representing breakpoints in the plots are considered threshold values which distinguish the anomalies in the different types of the ore deposit. In the C-A model, breakpoints between straight-line segments in the log–log plots present threshold values separating populations of concentration values to separate geochemical anomalies (Goncalves *et al.*, 2001; Afzal *et al.*, 2010). Threshold values in this are recognized by applying the fractal C-A model to demonstrate the boundaries between different anomalies (Cheng *et al.*, 1994; Shamseddin Meigoony *et al.*, 2014; Aramesh Asl *et al.*, 2015).

To calculate $A(\rho \leq \upsilon)$ and $A(\rho \geq \upsilon)$ enclosed by a concentration contour in a 2D model, the geochemical data of the ore element including corresponding concentrations are interpolated using the IDW and OK estimation methods.

Kriging is considered as a geostatistical algorithm for the estimation of different regional variable (e.g., ore element in this case) which is commonly described as a "minimum variance estimator", especially OK (Journel and Huijbregts, 1978; Davis, 2002; Bayraktar and Turalioglu, 2005; Emery, 2005; Li and Heap, 2008; Hormozi *et al.*, 2012; Shademan Khakestar *et al.*, 2013). The most common geostatistical method is OK which plays a special role because of its compatibility with a stationary model which involves a variogram (Chile's and Delfiner, 1999; Afzal *et al.*, 2011; Soltani Mohammadi *et al.*, 2012). Moreover, the OK estimates, which is a linear model with respect to local neighborhood structure, is based on a moving average of the variable of interest satisfying various dispersion forms of data (e.g., sparse sampling points; Goovaerts, 1997; Tahmasebi and Hezarkhani, 2010).

IDW is one of the commonly used methods for interpolation of scatter points in a block model. The IDW is based on the assumption that the interpolating surface should be influenced most by the nearby points and less by the more distant points. The interpolating surface is a weighted average of the scatter points and the weight assigned to each scatter point diminishes as the distance from the interpolation point to the scatter point increases. In this technique, the values to unknown points are calculated with a weighted average of the values available at the known points IDW. The IDW formula is expressed as follows (Journel and Huijbregts, 1978; Journel, 1983):

$$W_i = 1/d(X, X_i)^p \qquad (2)$$

where x denotes an interpolated (arbitrary) point, x_i is an interpolating (known) point, d is a given distance (metric operator) from the known point xi to the unknown point x, N is the total number of known points used in interpolation, and p is a positive real number, called the power parameter.

Discussion

Geological setting

The Irankuh district includes several sulfide and non-sulfide Zn–Pb deposits located in Irankuh Mountains, 20 km south of Esfahan (Central Iran) and belongs to the Sanandaj-Sirjan zone from the Zagros orogenic belt, as depicted in Figure 1. The Irankuh region comprises several Zn–Pb deposits, especially Goushfil (mainly sulfide ore) and Kolahdarvazeh (predominantly non-sulfide ore: Ghazban *et al.*, 1994; Reichert and Borg, 2008; Mokhtari *et al.*, 2015). The whole area shows an intensive and extensive faulting.

Jurassic Shemshak Formation shales have been overlain by a Cretaceous dolomitic and calcareous rocks (Ghazban *et al.*, 1994; Mokhtari *et al.*, 2015: Fig. 2). The Irankuh deposits are identified as Mississippi Valley Type (MVT) by Ghazban *et al.* (1994) and Reichert (2007), based on their discordant nature, emplacement along the Irankuh Fault, progressive depletion in stable C isotope ratios of the host dolomites, and isotopic characteristics of the ore-stage dolomites and barites. These deposits are stratabound and carbonate-hosted bodies with an ore mineral assemblage consisted of sphalerite, galena, pyrite, marcasite, dolomite, calcite, quartz, rarely barite, fluorite, celestine, gypsum, anhydrite, and pyrrhotite (Ghazban *et al.*, 1994; Reichert and Borg, 2008; Mokhtari *et al.*, 2015).

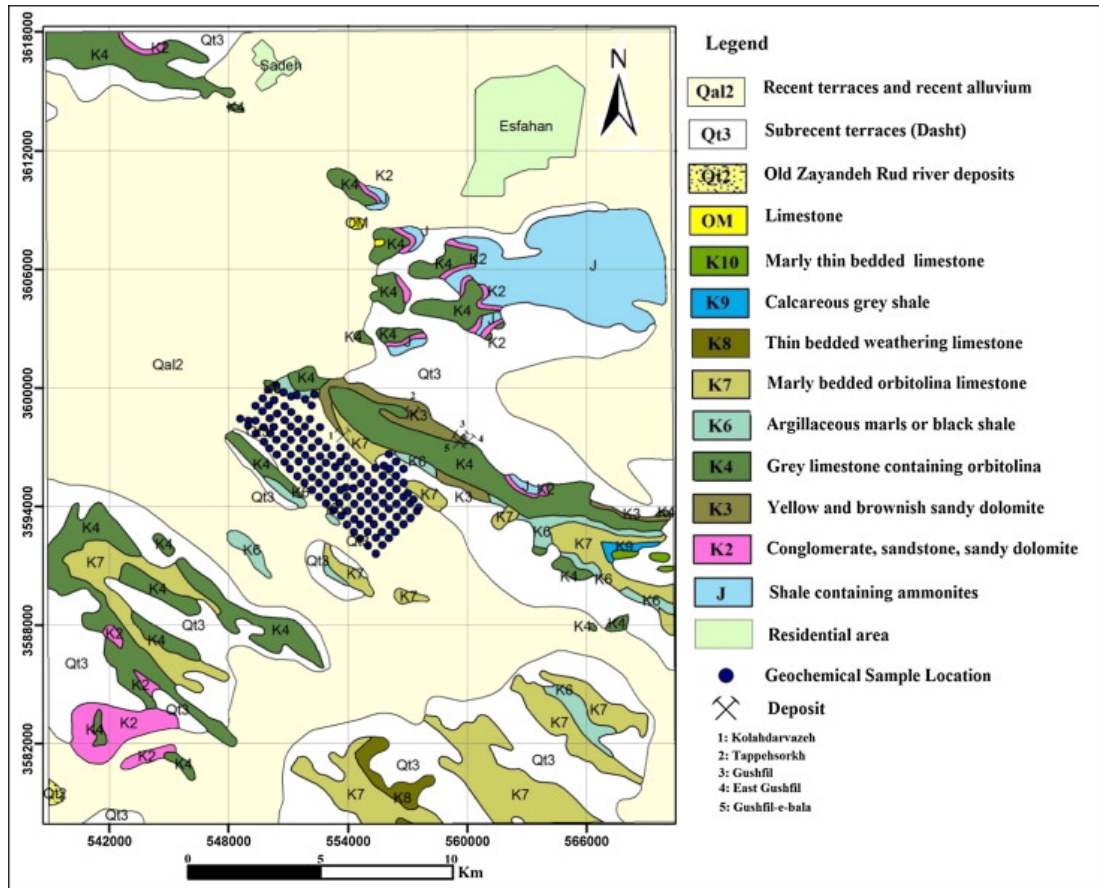Figu 1. Location of studied area in the map of Iran (Reichert and Borg, 2008)

Figure 2. Location of samples within geological map of the studied area (Mokhtari et al., 2015)

Variography and anisotropic ellipsoid

Variograms and anisotropic ellipsoids are a set of tools for spatial interpolation which are the critical components for geostatistical modeling (Verhoef and Cressie, 1993; Calder and Cressie, 2009). Based on 804 collected soil samples, the non-directional and directional variograms for Zn and Pb were generated using RockWorks software package, as shown in Figure 3. Consequently, anisotropic ellipsoids and the axis characteristics were also provided. This reveals that the anisotropic ellipsoids for Zn and Pb are similar in the studied area (Fig. 3).

Estimation by OK and IDW

Classical statistical parameters and histograms for Pb and Zn indicate that there are none-normal distributions (near to L distribution) with a positive skewness for both of them, as depicted in Figure 4. Based on the none-normal distributions, their medians are threshold values for Pb and Zn which are low values according to the high amounts of low grades for Pb and Zn (Table 1). Additionally, the classical methods based on summation of mean and standard deviation could not use because there are not normal distribution for Pb and Zn.
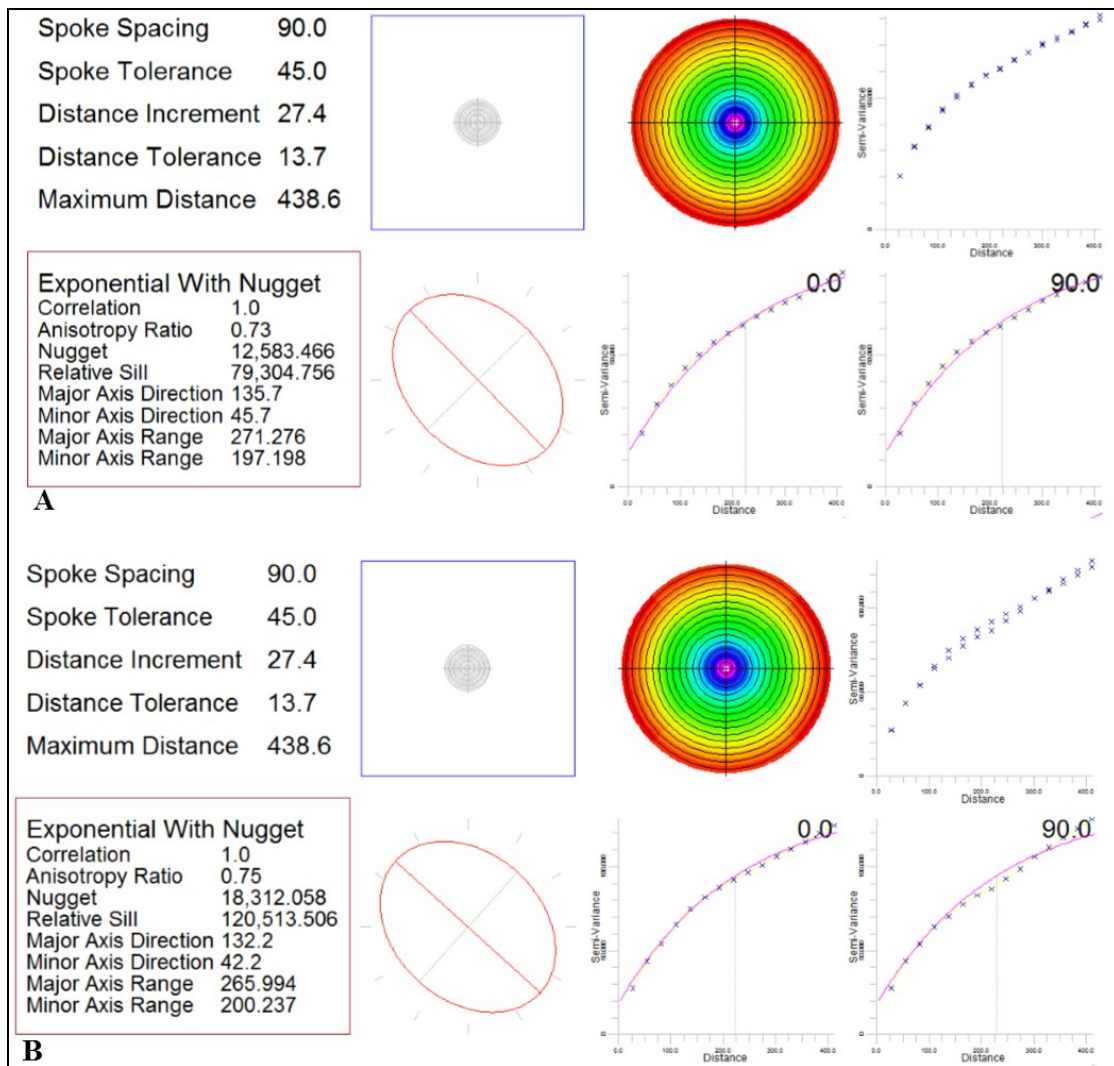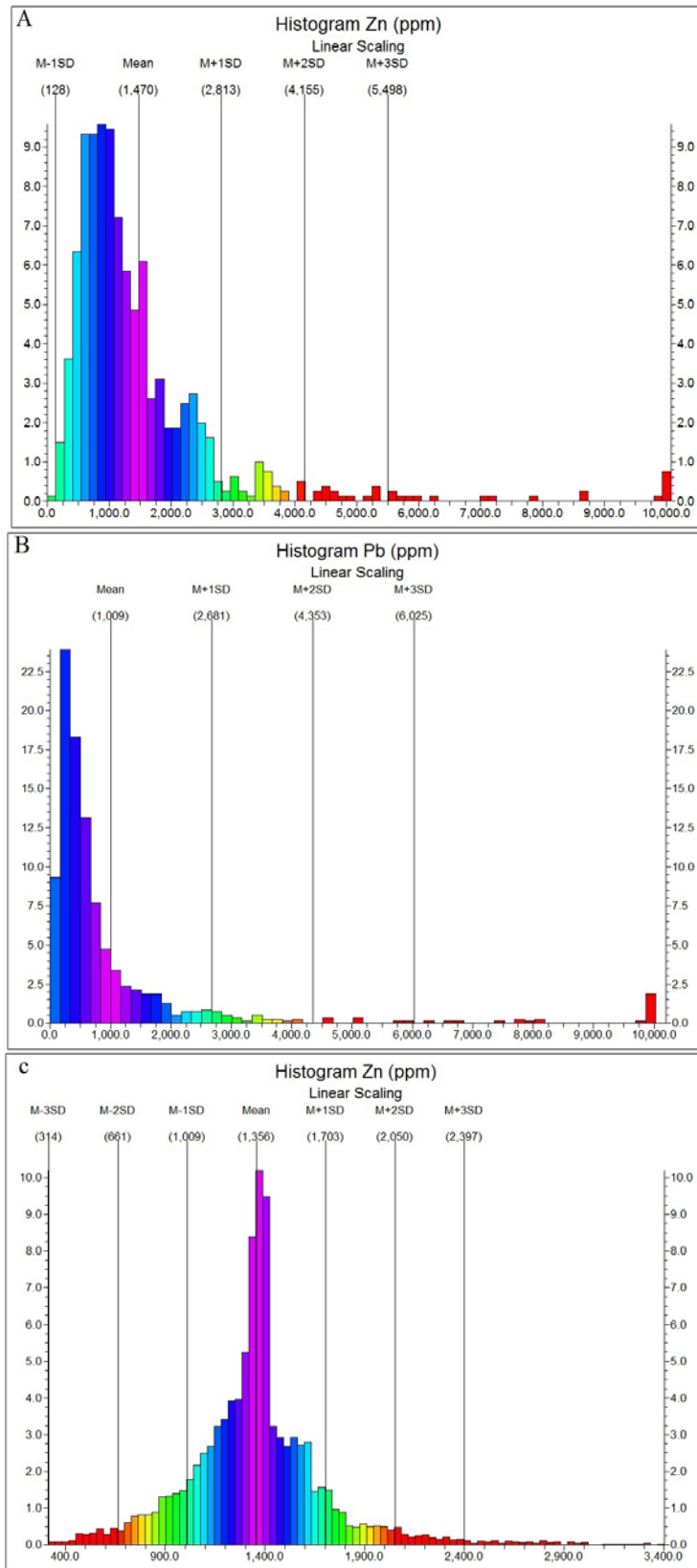

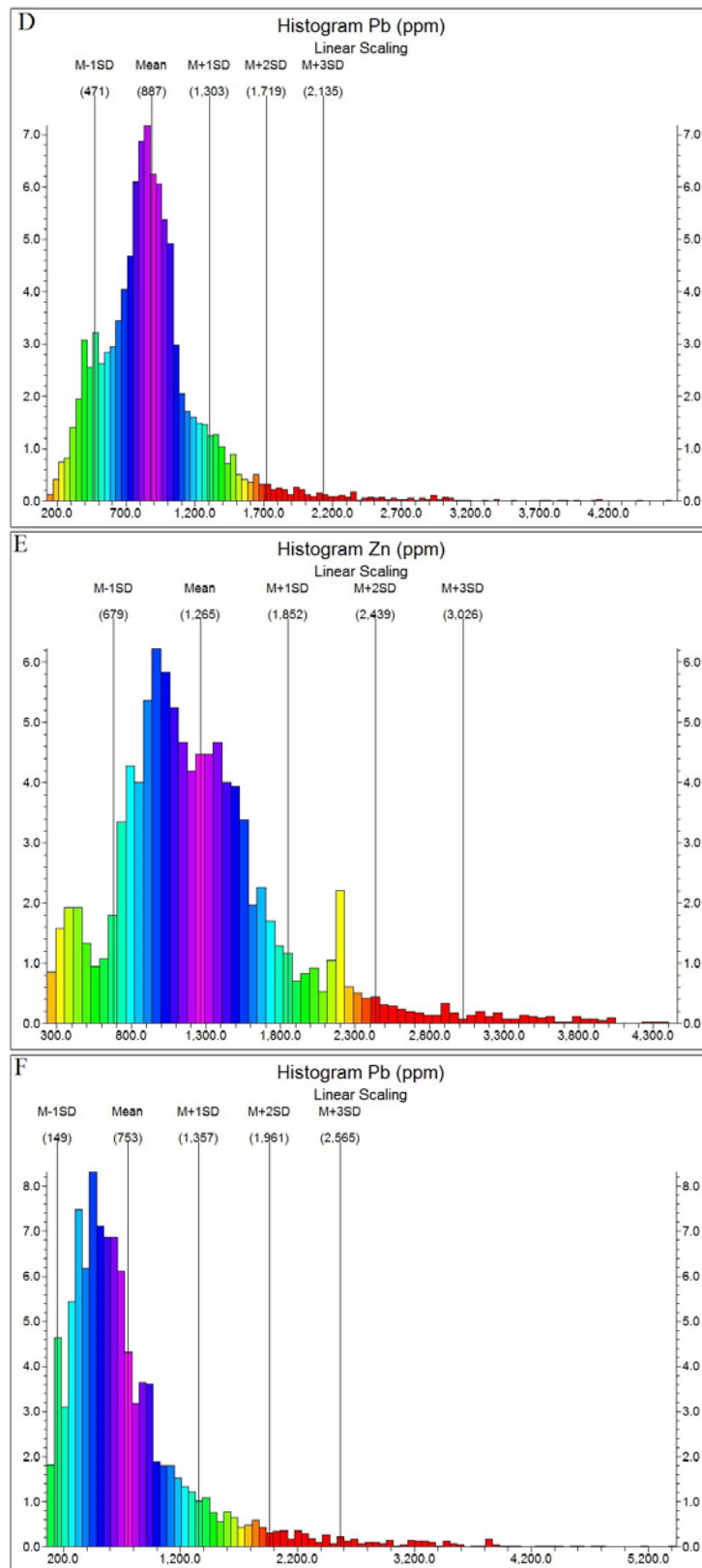Figure 3. Variograms and anisotropic ellipsoid for A: Zn and B: Pb

Figure 4. Histograms of A) Zn raw data B) Pb raw data C) Zn estimated data by OK D) Pb estimated data by OK E) Pb estimated data by IDW and F) Pb estimated data by IDW

Table 1. Statistical characteristics for Zn and Pb raw and estimated data

Raw data		
Statistical parameters	Zn (ppm)	Pb (ppm)
Mean	1470	1009
Median	1902.5	487
Variance	1802.44	2796.35
IDW Method		
Statistical parameters	Zn (ppm)	Pb (ppm)
Mean	1256	753
Median	1169.6	595.2
Variance	344236.46	364970.16
OK Method		
Statistical parameters	Zn (ppm)	Pb (ppm)
Mean	1356	887
Median	1355.5	844.7
Variance	120529.07	172947.8

Determination of the various cell dimensions within the 2D model is significant for geochemical exploration. David (1970) proposed general method for the operation according to the sampling pattern and ore deposit type. Therefore, cell dimensions are equal to 40 m×40 m for X and Y, respectively.

According to variography and anisotropic ellipsoid, the distribution models for Zn and Pb values in the deposit were built up by both OK and IDW methods. Results obtained by the both methods were compared based on their estimation variances and similarity to the used raw data for 804 collected soil samples (Fig. 4). Based on this comparison, variance of estimation derived via OK is lower than this obtained by the IDW method. Furthermore, mean values for Zn and Pb raw data have better correlation with their mean values obtained by the OK method, as depicted in Figure 4 and Table 1.

C-A fractal modeling
Based on the IDW and OK models for Zn and Pb, areas corresponding to the different grades were calculated to derive a C-A fractal model. Threshold values for Zn and Pb were recognized in the C-A log-log plots for both methods (Fig. 5) which revealed a power-law relationship between their concentrations and areas occupied.
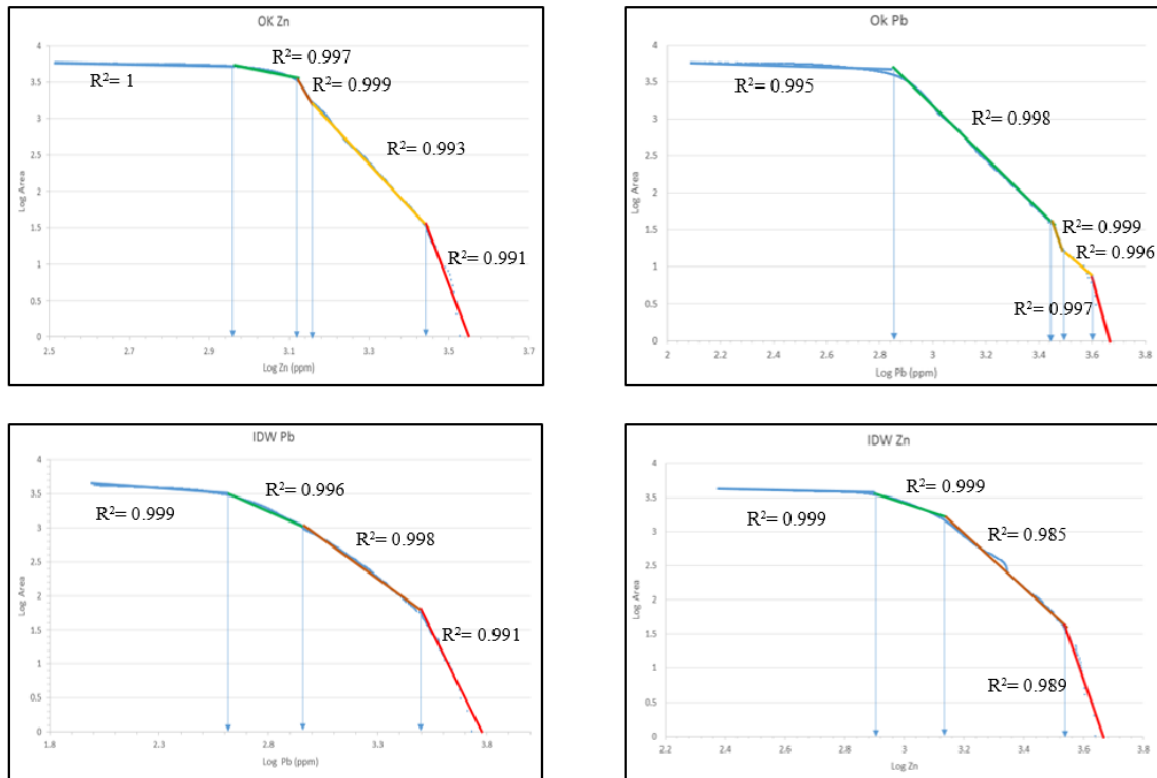

Figure 5. C–A log–log plot for Zn and Pb concentrations based on OK and IDW methods

The depicted arrows in the log-log plots show five threshold values (breakpoints) corresponding to 912 ppm, 1318 ppm, 1445 ppm, and 2754 ppm for estimated Zn by OK method and three thresholds including 794 ppm, 1380 ppm, and 3467 ppm for estimated Zn via IDW method. However, four and three thresholds were determined due to OK and IDW methods for Pb, respectively. For each method based on the log-log plots, geochemical anomalies were separated into four distinct categories called extremely, highly, moderately, and weakly anomalies, as depicted in Table 2. Threshold for extremely anomaly are similar for Pb in the both method (≥ 3162 ppm).

According to the C-A fractal model, most of the extremely anomalies for Zn are situated in the NW part of the area resulted by both methods (Fig. 6).

Table 2. Separation of anomalies in the Irankuh area based on thresholds of Zn and Pb defined from the C–A fractal model

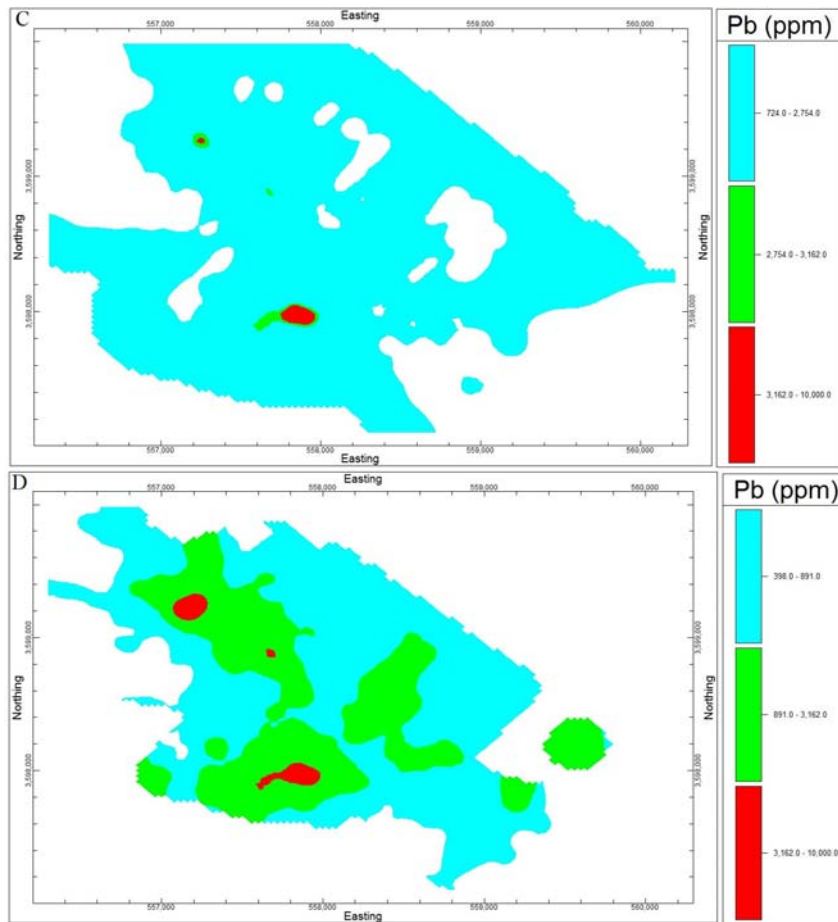| OK Method		
Anomaly	Zn (ppm)	Pb (ppm)
Extremely	≥ 2754	≥ 3162
Highly	1318-2754	2754-3162
Moderately	912-1318	724-2754
Weakly	≤ 912	≤ 724
IDW Method		
Anomaly	Zn (ppm)	Pb (ppm)
Extremely	≥ 3467	≥ 3162
Highly	1380-3467	891-3162
Moderately	794-1380	398-891
Weakly	≤ 794	≤ 398
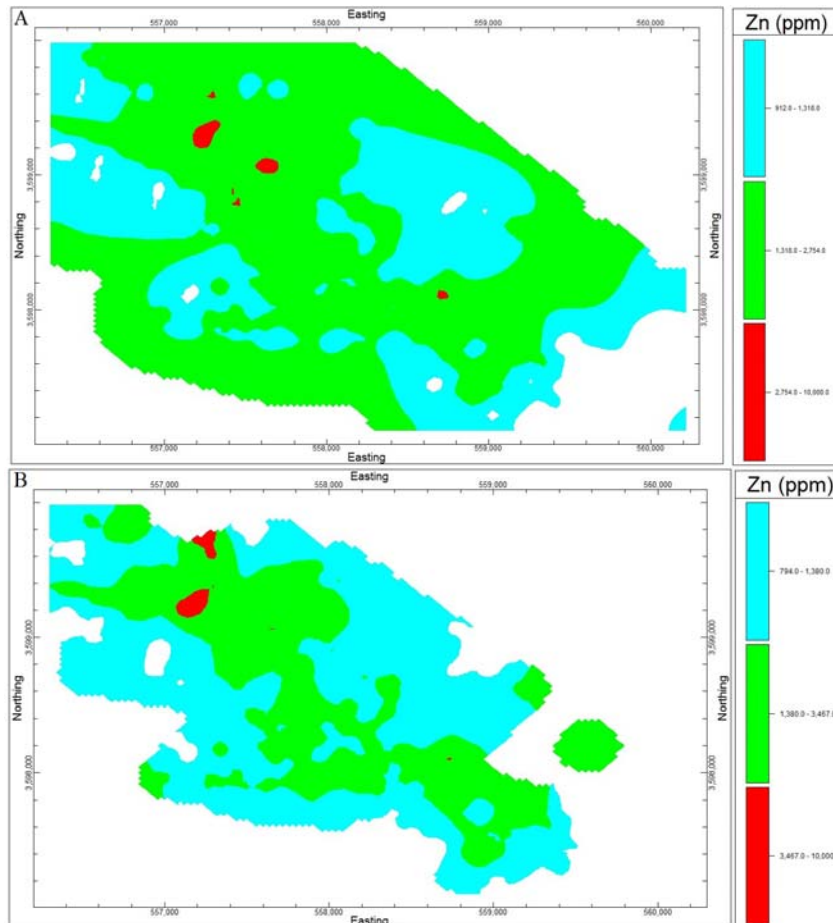
Figure 6. Separation of anomalies by the C–A fractal model a) OK method for Zn b) IDW method for Zn c) OK method for Pb and d) IDW method for Pb

Furthermore, Pb extremely anomalies are located in the NW and central parts of the area based on the both method, as depicted in Figure 6.

Comparison between OK and IDW variances in the fractal modeling
The variances of different Zn and Pb geochemical anomalies derived via fractal modeling demonstrate that variances obtained by the OK are lower than IDW for Zn (Table 3). Variances derived via the OK are lower than those for IDW in the extremely and highly Pb anomalies, however, the values of variances for other Pb anomalies are lower on the basis of IDW for moderately and weakly Pb anomalies, as depicted in Table 3.

Table 3. Variances of Zn and Pb mineralized zones obtained by the C-V modeling based on the IDW and OK estimation data

Mineralized zone	OK variance for Zn	IDW variance for Zn	OK variance for Pb	IDW variance for Pb
Extremely	34323.20	58071.35	178585.04	288843.63
Highly	59875.33	194838.58	9905.78	245259.33
Moderately	12358.96	26795.06	96152.29	16999.01
Weakly	20564.87	28949.79	19574.97	7622.71

As a result, the C-A fractal modeling based on the estimated data from the OK estimation method has higher accuracy compared to the IDW estimated data.

Conclusion
Choosing the appropriate method for estimation with the minimum variance is important in any classical statistics and fractal modeling. Most of the

classical statistics and fractal/multifractal models are defined based on estimated data. In this study, the 2D grid models from OK and IDW interpolation methods were built up from soil geochemical data. The variances distribution in both methods were examined for the Irankuh area considering the C-A fractal model. Estimation variance for the OK method is less than that in the IDW for extremely, highly, moderately and weakly Zn anomalies. Based on Pb anomalies obtained by the C-A fractal modeling, the moderately and weakly anomalies estimated by the IDW method have better accuracy, however, extremely and highly anomalies resulted

by OK have less variances. Main Zn-Pb prospects are located in the NW part of the area. As a result, none-normal distributions for Pb and Zn and a dense sampling grid show that the fractal modeling is proper for separation of geochemical anomalies in the studied area.

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